ARL 65-84
MAY 1965

Aerospace Research Laboratories

HEAT TRANSFER IN CYLINDRICAL PIPES WITH TURBULENT FLOW AND ARBITRARY WALL FLUX AND TEMPERATURE DISTRIBUTIONS

JOHN W. GORESH
FLUID DYNAMICS FACILITIES LABORATORY

OFFICE OF AEROSPACE RESEARCH

United States Air Force

NOTICES

300 - July 1965 - 448-48-1035

ARL 65-84

HEAT TRANSFER IN CYLINDRICAL PIPES WITH TURBULENT FLOW AND ARBITRARY WALL FLUX AND TEMPERATURE DISTRIBUTIONS

JOHN W. GORESH

FLUID DYNAMICS FACILITIES LABORATORY

MAY 1965

Project 7065

AEROSPACE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FOREWORD

This technical report was prepared by John W. Goresh, Fluid Dynamics Facilities Laboratory, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, on Project 7065, "Aerospace Simulation Techniques Research," under the direction of Mr. Elmer G. Johnson, Chief of the Laboratory.

The author is particularly grateful to Dr. Robert G. Dunn, Senior Scientist, Fluid Dynamics Facilities Laboratory, for his guidance and encouragement during the preparation of this report. Also, the author wishes to extend his gratitude to Henry E. Fettis and James C. Caslin of the Applied Mathematics Research Laboratory for their helpful suggestions in formulating the analytical expressions and adapting these to numerical computations. Especially appreciated was the assistance of Norman E. Scaggs, Project Engineer on the ARL 30-inch hypersonic wind tunnel, for furnishing the experimental data needed for the final computations.

ABSTRACT

The problem considered in this paper is that of finding the heat losses from a turbulent gas flowing through a pipe due to free and forced convection, conduction through the solid insulation and external wall radiation. Furthermore, along the inner wall, neither the heat flux nor the temperature variation is known in advance, but are determined by the losses from the outer wall. The analytical procedure used here is analogous to that first used by Latzko (1) and later modified by Fettis (7) for a constant wall temperature, where eigenfunctions of an approximate equation are first obtained and these are then used to obtain the solution to the actual heat equation.

The numerical solution for the case $\alpha = 0.7$ is presented in the form of graphs of axial temperature distribution along the pipe as well as radial temperature profiles within the gas stream. The case where the heat transfer parameter $\alpha = 0.7$ is used corresponds to the specific dimensions and thermal properties of the stagnation chamber insulation in the ARL 30-inch hypersonic wind tunnel. This tunnel can be briefly described as a blow-down type facility which uses a methane-oxygen fired zirconia pebble bed which heats the air to the neighborhood of $4400^{\circ}R$ prior to expansion to flow Mach numbers ranging from 16 to 22 at free stream Reynolds numbers approaching 10^6 per foot. The axial temperature distributions and radial profiles presented herein are for a specific set of conditions under which the ARL 30-inch hypersonic wind tunnel has actually operated.

TABLE OF CONTENTS

LIST OF FIGURES

NOMENCLATURE

A	constant defined by equation (4)
A_n	constants determined from the inlet condition
a	parameter used in the hypergeometric equation (27)
b	parameter used in the hypergeometric equation (27)
C_p	specific heat
c	parameter used in the hypergeometric equation (27)
$g_n(z)$	eigenfunctions for equation (21-a)
$\bar{h}$	combined heat transfer coefficient for free convection and external wall radiation
h_c	heat transfer coefficient due to free convection
h_g	heat transfer coefficient of the gas in the inner tube
h_o	heat transfer coefficient at the inlet to the channel, expressed as unit thermal conductance by Latzko
k_{eff}	effective over all conduction coefficient of the pipe insulation
k_c	thermal conductivity of the gas some distance away from the outer surface of the pipe
k_g	thermal conductivity of the hot gas in the inner tube
N	heat transfer parameter given by equation (11)
n	exponent in equation (4), depending on the Reynolds number range
Pr	Prandtl number
p	parameter defined by equation (9-a)
q	total heat flux
q_a	heat flux of air tangent to the outer wall
q_o	heat flux at pipe inlet
q_R	heat flux along the radius of the inner wall surface

q_r	radial heat flux $(\frac{BTu}{hr\text{-}ft^2})$, positive in the + r direction
q_{gw}	heat flux of gas designated as region (2) in figure 6
q_{s_1}	heat flux designated as region (1) in figure 6
q_{s_2}	heat flux due through the solid layers from the point tangent to the outer layer of the gas
R	radius of the inner tube surface
Re	Reynolds number
r	radial coordinate
r_2	radius from center to inner wall surface as indicated in figure 6
r_1	radius from center to outer wall surface
$\bar{r}$	nondimensional radius
s	subscript s refers to solid portion of tube, where s and s_2 refer to outer and inner solid surface layers respectively
T	temperature
T_a	temperature of the gas immediately tangent to the outer wall
T_c	temperature of gas at the centerline of the tube.
T_{gw}	temperature of the gas at the inner wall designated as region 2 in figure 6
T_1	temperature at the outer solid wall of the cylinder
T_2	temperature of the solid wall at any point immediately adjacent to the gas at the wall
T_{ref}	reference temperature of the environment
U_R	effective thermal conductance considered at the radius of the inner tube surface
u	velocity of the fluid stream in the direction of flow
V	average velocity
x	axial coordinate
$\bar{x}$	nondimensional axial coordinate

α	heat parameter defined as $\frac{2}{7N}$
β_n	parameter which is proportional to the exponential decay factor for the temperature in the axial direction
Γ	refers to gamma function
η	ratio of the conduction to radius as defined by equation (7-6) under section VII
θ	temperature difference between the local temperature and a reference temperature
θ_o	temperature difference at the inlet of inner tube
θ_m	mean temperature difference
λ_n	eigenvalues for equation (23)
ν	kinematic viscosity
ρ	density
$\sum$	referring to a summation
τ	local shear stress
τ_R	inner wall shear stress
Φ_n	eigenfunctions for the approximate differential equation, that is, corresponding to the eigenvalues expressed by λ
ω_n	eigenvalues for the actual heat equation (21-a)

I. INTRODUCTION

The problem of heat transfer in fully-established turbulent flow in a cylindrical tube has been investigated by many researchers (Reference 1 through 4). In all cases the flow Reynolds number is sufficiently large to justify the assumption of negligible axial conduction in the fluid. As a consequence, the mathematical investigations were reduced to solving an eigenvalue problem with the wall temperature taking the form of a step function. Once this was accomplished, other boundary conditions such as prescribed heat flux or prescribed wall temperature variation were included by the method of superposition, an idea which was an outgrowth of Duhamel's theorem. This was, to the author's knowledge, first pointed out by Tribus. However, there are certain situations in which the use of the superposition integral may not be practical. Such is the case in a cross-flow heat exchanger when the wall heat transfer rate is related to an external heat transfer coefficient. In this case, neither the heat flux nor the temperature variation is known in advance and the flux must be matched with the wall temperature variation. To accomplish this one needs to know the eigensolutions and the coefficients quite accurately near the entrance.

The practical engineering use for the steady state heat transfer analysis and numerical example presented here is that of determining the heat losses from a turbulent gas flowing through an insulated pipe. A cross-section of the pipe considered in this study is shown in Figure 1. Consideration is given to finding heat losses due to all three modes of heat transfer with a further stipulation that the surface temperature of the chamber is not held constant.

In the first portion of the paper, consideration is given to solving the general heat transfer problem for turbulent fluid flow through cylindrical tubes. The analytical approach for solving the problem is given in the

Manuscript released by the author April 1965 for publication as an ARL Technical Report.

following three sections. The remainder of the paper gives a numerical example for the specific case corresponding to the various thermal conditions in the stagnation chamber of the ARL 30-inch hypersonic wind tunnel.

II. STATEMENT OF THE PROBLEM

The problem is stated as follows: given the initial temperature distribution (with respect to the radial distance) of a fluid entering a cylindrical tube through which heat transfer occurs by means of a fixed wall conductance, the ensuing temperature distribution is to be determined.

Since the accent of the investigation is on this particular boundary condition, the conventional assumptions of constant-property (including density) fluid are adopted. The flow Reynolds number is in the order of 10^5 and the length-diameter ratio is large so that very little error is introducted by assuming the existance of fully-established flow beginning at the entrance section. Ignoring axial conduction, the steady-state energy equation with circular symmetry is

$$- \rho \; C_p \; u \; \frac{\partial T}{\partial x} = \frac{1}{r} \; \frac{\partial}{\partial r} \; (r \, q_r) \tag{1}$$

where q_r is the radial heat flux, positive in the + r direction. On the inner wall r = R, the flux is equal to the thermal conductance U_R multiplied by the temperature difference between the inner wall and a reference temperature, T_{ref}. Thus, at r = R:

$$q_R = U_R \; (T - T_{ref}) \quad . \tag{2}$$

U_R includes contributions from the wall-thickness, convection and external wall radiation effects. It is assumed that U_R is effectively independent of axial distance x. At the entrance, the fluid temperature has its initial value θ_o (r), that is at x = o

$$\theta = \theta_o \; (r) \, . \tag{3}$$

The equations (1), (2), and (3) for "q" constitute the present problem.

III. ANALYTICAL METHODS

Currently there are four different approaches toward the solution of the stated problem. They can be classified into two main categories; (i) eigenvalue approach represented first by the work of Latzko (1) and later modified by Fettis (7), Sleicher and Tribus (2) and Beckers (3),and (ii) boundary layer approach typified by the work of Deisler (4) and Sparrow (5). As a general conclusion, the analysis of (4) and (5) based on the integral method of the boundary layer equations gives a gross picture without a high degree of precision in its numerical results. Among the eigenvalue approach techniques, there are three different ways of handling the turbulent velocity distribution and the eddy diffusivity distribution. Latzko employed a simplified 1/7 power law closed form distribution for the velocity and the total thermal diffusivity (turbulent plus molecular). Sleicher and Tribus used experimentally measured velocity distributions and eddy diffusivities. This resulting procedure was to solve the problem for each Reynolds number and Prandtl number.

To use the method given by Beckers, one needs to divide the flow into three different regimes; viscous sublayer, buffer layer and turbulent core. In fact, he solved three sets of differential equations with three solutions which are joined together with temperature continuity only.

The approach of Sleicher and Tribus appears to be the best procedure, but the numerical work for a general treatment is also the most time-consuming. The second best is the Latzko method which can be modified for a different Reynolds number range by using different power-law exponents in his original analysis. In view of the flexibility afforded by the Latzko's formulation, a portion of the latter approach is adopted in the present problem.

IV. ANALYSIS

Latzko's analysis is based on the premise of the power-law distribution of velocity near the wall which works quite well in that vicinity. However, since the velocity profile becomes relatively flat near the tube axis, he expressed the velocity distribution with the following expression,

$$u = A \left\{ \frac{R^2 - r^2}{2R^2} \right\}^{\frac{1}{n}} \tag{4}$$

Equation (4) reduces to $u \sim \delta^{\frac{1}{n}}$ when r is close to R, that is $\delta = R - r \approx 0$. The coefficient "A" is related to the mean flow by a constant factor dependent on n.

For fully established flow, the total (molecular and eddy) shear is a linear function of the radius. If equation (4) is differentiated, one obtains the following,

$$\frac{\partial u}{\partial r} = \frac{A}{n} \left(\frac{R^2 - r^2}{2R^2} \right)^{\frac{1-n}{n}} \left(\frac{-r}{R^2} \right)$$

or (5)

$$- \frac{\partial u}{\partial r} \left(\frac{nR}{A} \right) \left(\frac{R^2 - r^2}{2R^2} \right)^{\frac{n-1}{n}} = \frac{r}{R}$$

In general n can take on values of 6, 7, 8 and 9 depending on the various Reynolds number ranges considered; hence the velocity derivative exhibits a singularity at r = R, indicative of the fact that the power law expression cannot be used at the wall. However, the quantity on the left hand side of equation (5) remains regular at r = R which must be equal to the shear ratio (τ/τ_R), where τ_R is the wall shear. Accordingly, Latzko expressed the shear ratio as,

$$\frac{\tau}{\tau_R} = \frac{-nR}{A} \left(\frac{r^2 - R^2}{2R^2}\right)^{\frac{n-1}{n}} \frac{\partial u}{\partial r}$$

Further, he used the Reynolds analogy for a Prandtl number equal to one and obtained an expression for the heat flux using $n = 7$,

$$q_r \approx - \text{const.} \left(\frac{R^2 - r^2}{2R^2}\right)^{6/7} \frac{\partial \theta}{\partial r} \tag{6}$$

In this manner the temperature derivative is permitted a singularity at $r = R$, but the heat flux as defined by (6) is regular at $r = R$. Quantitatively, Latzko used the following expression for heat flux,

$$q_r = \frac{-0.199\, V^{3/4}\, \nu^{1/4}\, \rho\, C_p}{(2R)^{3/28}} \left(\frac{R^2 - r^2}{2R}\right)^{6/7} \frac{d\theta}{dr} \tag{6-a}$$

and in the presently used notation, q_r is expressed as

$$q_r = -\left(\frac{0.199}{2^{12/7}}\right) k_g \, Pr \, Re^{3/4} \left[1 - \left(\frac{r}{R}\right)^2\right]^{6/7} \frac{\partial \theta}{\partial r} \tag{7}$$

With the constant A in equation (4) evaluated for $n = 7$, the velocity is expressed as

$$u = \frac{8}{7} V \left(\frac{1 - r^2}{R^2}\right)^{1/7} \tag{8}$$

Introducing the non-dimensional variables $\bar{r} = \left(\frac{r}{R}\right)$ $\bar{x} = \left(\frac{x}{r}\right)$ and the temperature difference as θ, equation (1) reduces to

$$p\,(1 - \bar{r}^2)^{1/7} \frac{\partial \theta}{\partial \bar{x}} = \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left[\bar{r}\; (1 - \bar{r}^2)^{6/7} \frac{\partial \theta}{\partial \bar{r}} \right] \tag{9}$$

$$\text{where } p = \left(\frac{8 \times 2^{5/7}}{7 \times 0.199} \right) Re^{1/4} \tag{9-a}$$

The boundary condition at $r = R$ is $q_R = U_R\,(T - T_{ref})$ or

$$\left\{ - \frac{0.199}{2^{1\,1/7}}\; k_g\, P_r\, R_e^{3/4} \left[1 - \bar{r}^2 \right]^{6/7} \frac{1}{R} \right\} \frac{\partial \theta}{\partial \bar{r}} = U_R\,(T - T_{ref})$$

for $\bar{r} = 1$,

$$\left\{ (1 - \bar{r}^2)^{6/7} \frac{\partial \theta}{\partial \bar{r}} \right\}_{\bar{r} = 1} + N\theta = 0 \tag{10}$$

$$\text{where } N = \left(\frac{U_R\, R}{k_g} \right) Re^{-3/4} \left(\frac{2^{1\,1/7}}{0.199} \right) . \tag{11}$$

The temperature distribution at $\bar{x} = 0$, becomes

$$\theta = \vartheta_0\,(\bar{r}) \tag{12}$$

where θ_0 represents the temperature difference at the inlet of the pipe. A second condition must be satisfied which demands that the centerline temperature be finite. This second condition will be given after another change of variables is made and substituted into equation (9).

A solution to equation (9) is assumed to be of the form,

$$\theta = \exp\left(\frac{-\beta^2 \bar{x}}{p} \right) g\,(\bar{r}) \tag{13}$$

Substitution of equation (13) into equation (9) yields the following

$$-\beta^2 \, (1 - \bar{r})^{1/7} \, g = \frac{1}{\bar{r}} \frac{d}{d\bar{r}} \left[\bar{r} \, (1 - \bar{r}^2)^{6/7} \frac{dg}{d\bar{r}} \right] \tag{14}$$

Introducing the change of variables

$$z^7 = 1 - \bar{r}^2 \tag{15}$$

equation (14) becomes

$$\frac{d}{dz} \left[(1 - z^7) \frac{dg}{dz} \right] + \left(\frac{49\beta^2}{4} \right) z^7 \; g = 0 \tag{16-a}$$

and equation (10) is written as:

$$- \frac{2}{7N} \left. \frac{dg}{dz} \right|_{z = 0} + g = 0 \tag{16-b}$$

The second condition, mentioned previously, is to satisfy equation (16-a) at $z = 1$ or the centerline. Because of the singularity at $z = 1$, it is necessary to require that g be finite at $z = 1$. The system described by (16-a) with proper boundary conditions is a regular Sturm-Liouville system solved as an eigenvalue problem. The o thogonality relation given below is readily established,

$$\int_0^1 z^7 \, g_n(z) \, g_m(z) \, dz = 0 \quad \text{for } m \neq n \tag{17}$$

The initial conditions are satisfied by putting

$$\theta = \sum A_n \exp\left(\frac{-\beta_n^2}{p} \bar{x}\right) g_n\left(z\right) , \qquad (18)$$

where the coefficients A_n are determined from

$$A_n = \frac{\int_o^1 z^7 \theta_o g_n (z) dz}{\int_o^1 z^7 g_n^2 (z) dz} \qquad (19)$$

and θ_o is defined at $\bar{x} = o$, or $\theta_o = \theta (z, o)$.

$$\text{Let } \omega_n = \frac{49}{4} \beta_n^2 ; \qquad (20)$$

then equations (16-a) and (16-b) respectively reduce to

$$\frac{d}{dz}\left[(1 - z^7) \frac{dg}{dz}\right] + \omega z^7 g = o \qquad (21\text{-a})$$

Evaluating at $z = o$, and for convenience in later computing the equation involving gamma functions, letting $\frac{2}{7N} = \alpha$, we have

$$\alpha \left(\frac{dg}{dz}\right)\Bigg]_{z = o} + g\Bigg]_{z = o} = o \qquad (21\text{-b})$$

Solutions of (21-a), which satisfy the condition stipulated at $z = 1$ and also of equation (21-b) will exist only for discrete values of the parameter ω as indicated in equation (21-a). Once these eigenvalues have been found, the complete solution of equation (9) will then be of the form

$$\sum A_n g_n (z) \exp \left(- \frac{\omega_n}{p} \quad \bar{x} \right) \qquad (22)$$

Before considering the solution of (21-a), we wish to consider the solution of an auxiliary equation which is similar in appearance, although not identical, to equation (21-a), that is:

$$\frac{d}{dz} \left[(1 - z^7) \frac{dg}{dz} \right] = - \lambda z^5 g \qquad (23)$$

A change of variable $z^7 = t$ converts equation (23) to the following

$$t (1 - t) \frac{d^2 g}{dt^2} + \left(\frac{6}{7} - \frac{13 t}{7} \right) \frac{dg}{dt} + \frac{\lambda}{49} g = o \qquad (24)$$

A similar equation results if we consider still another change of variables, this being

$$y = 1 - t \qquad \text{and} \qquad dy = -dt \qquad (25)$$

Then equation (24) becomes

$$y (1 - y) \frac{d^2 g}{dy^2} + \left(1 - \frac{13}{7} y \right) \frac{dg}{dy} + \frac{\lambda g}{49} = o \qquad (26)$$

Equations (24)and (26) are in the form of the hypergeometric equation

$$y (1 - y) \frac{d^2 g}{dt^2} + \left[c - (a + b + 1) y \right] \frac{dg}{dy} - a b g = o \qquad (27)$$

Equation (24) is useful when obtaining solutions for $\alpha = o$ and ∞. When $\alpha = o$, then $\theta = o$ which corresponds to Latzko's solution to the heat transfer

problem, also, when $\alpha = \infty$, the solution corresponds to the case $q_R = o$. For the case of $q_R = o$, a meaningful expression for the temperature at $\bar{x} = o$ must be prescribed, that is, θ_o cannot be independent of the radius. Also, a solution is sought such that the initial inlet expression for θ_o is successively "damped out." For all other values of the parameter "α", we must obtain solutions to equation (26).

With the aid of equation (27), comparison of coefficients a, b and c can be made which are used in equations (24) and (26). For equation (24) we obtain the following;

$$\begin{aligned} ab &= \frac{-\lambda}{49} \\ b &= \frac{6}{7} - a \\ c &= \frac{6}{7} \end{aligned} \tag{28}$$

while for equation (26), we obtain

$$\begin{aligned} ab &= \frac{-\lambda}{49} \\ b &= \frac{6}{7} - a \\ c &= 1 \end{aligned} \tag{29}$$

For the general case, equation (27), which includes $\alpha = o$ and $\alpha = \infty$, two linearly independent solutions are

$$g_1 = F(a, b, 1, y) \tag{30-a}$$

$$g_2 = F_2(a, b, 1, y) \ln y \tag{30-b}$$

but because of the requirement that g be finite at z = 1, only the solution (30-a) is admissible. Hence, except for a multiplicative factor, the desired solution of (27) is

$$g = F\left(a, \frac{6}{7} - a, 1, y\right) \tag{31}$$

The value of "a", and therefore λ, is still undetermined, and must be so chosen that the boundary conditions are satisfied.

From the known formula for the derivative of the hypergeometric function called "F"

$$\frac{dF}{dy} = \frac{ab}{c} F(a + 1, b + 1, c + 1, y) \tag{32}$$

the derivative of (31) can thus be written as

$$\frac{dg}{dy} = a\left(\frac{6}{7} - a\right) F\left(a + 1, \frac{13}{7} - a, 2, y\right) \tag{33}$$

At y = 1, $\frac{dg}{dy}$ is singular. However, the boundary condition is given in terms of $\frac{dg}{dz}$, and since $y = 1 - z^7$, we find that

$$\frac{dg}{dz} = \frac{dg}{dy}\frac{dy}{dz} = -7z^6 \frac{dg}{dy} \tag{34}$$

Equation (33) can now be written as

$$\frac{dg}{dy} = a\left(\frac{6}{7} - a\right) F\left(a + 1, \frac{13}{7} - a, 2, 1 - z^7\right) \tag{35}$$

Now the appropriate general expression for the funᵔtion F is the following:

$$F(a, b, c, y) = \frac{\Gamma(c)\,\Gamma(c - a - b)}{\Gamma(c - a)\,\Gamma(c - b)} F(a, b, a + b - c + 1, 1 - y)$$

$$+ (1 - y)^{c-a-b} \frac{\Gamma(c)\,\Gamma(a + b - c)}{\Gamma(a)\,\Gamma(b)} F(c - a, c - b, c - a - b + 1, 1 - y) \tag{36}$$

where $y = 1 - z^7$.

Hence,

$$F\left(a + 1, \frac{13}{7} - a, 2, y\right) = \frac{\Gamma(2)\,\Gamma(-\frac{6}{7})}{\Gamma(1 - a)\,\Gamma(\frac{1}{7} + a)} F\left(a + 1, \frac{13}{7} - a, \frac{13}{7}, z^7\right)$$

$$+ z^{-6} \frac{\Gamma(2)\,\Gamma\left(\frac{6}{7}\right)}{\Gamma(a + 1)\,\Gamma\left(\frac{13}{7} - a\right)} F\left(1 - a, \frac{1}{7} + a, \frac{1}{7}, z^7\right) \tag{37}$$

Substituting (37) into (35), one obtains the following:

$$\frac{dg}{dy} = a\left(\frac{6}{7} - a\right)\left\{\frac{\Gamma(2)\,\Gamma(-\frac{6}{7})}{\Gamma(1 - a)\,\Gamma(\frac{1}{7} + a)} F\left(a + 1, \frac{13}{7} - a, \frac{13}{7}, z^7\right)\right.$$

$$\left. + z^{-6} \frac{\Gamma(2)\,\Gamma(\frac{6}{7})}{\Gamma(a + 1)\,\Gamma(\frac{13}{7} - a)} F\left(1 - a, \frac{1}{7} + a, \frac{1}{7}, z^7\right)\right\} \tag{38}$$

But

$$\frac{dg}{dz} = \frac{dg}{dy}\,\frac{dy}{dz} = -7z^6\,\frac{dg}{dy}$$

hence,

$$\lim_{z \to o} \frac{dg}{dz} = -7a\left(\frac{6}{7} - a\right) \frac{\Gamma(2)\,\Gamma\left(\frac{6}{7}\right)}{\Gamma(a+1)\,\Gamma\left(\frac{13}{7} - a\right)}\, F\left(1 - a, \frac{1}{7} + a, \frac{1}{7}, o\right) \tag{39}$$

At the origin,

$$F\left(1 - a, \frac{1}{7} + a, \frac{1}{7}, 0\right) = 1$$

Therefore

$$\lim_{z \to o} \frac{dg}{dz} = \frac{-7\,\Gamma(2)\,\Gamma\left(\frac{6}{7}\right)}{\Gamma(a)\,\Gamma\left(\frac{6}{7} - a\right)} \tag{40}$$

At $z = o$, equation (31) becomes

$$g(o) = F\left(a, \frac{6}{7} - a, 1, 1\right) \tag{41}$$

where

$$F(a, b, c, 1) = \frac{\Gamma(c)\,\Gamma(c - a - b)}{\Gamma(c - a)\,\Gamma(c - b)} \tag{42}$$

We now obtain from equation (21-b), with the aid of equations (41) and (42), the relation

$$\frac{7a\,\Gamma\left(\frac{6}{7}\right)}{\Gamma(a)\,\Gamma\left(\left(\frac{6}{7} - a\right)\right)} = \frac{\Gamma\left(\frac{1}{7}\right)}{\Gamma(1 - a)\,\Gamma\left(\frac{1}{7} + a\right)} \tag{43}$$

Thus admissible solutions occur when "a" is such that

$$\alpha = \frac{\Gamma(\frac{1}{7})\,\Gamma(a)\,\Gamma(\frac{6}{7} - a)}{7\,\Gamma(1 - a)\,\Gamma(\frac{1}{7} + a)\,\Gamma(\frac{6}{7})} \tag{44}$$

The values for λ corresponding to values of "a" which satisfy equation (44) can then be obtained from the formula

$$\lambda = -49\,ab = -49a\,(\frac{6}{7} - a) \tag{45}$$

For values of α other than zero or infinity, the eigenfunctions of equation (23) may be obtained in the following form from use of equation (31):

$$g_n(z) = A_n\,F(a_n, \frac{6}{7} - a_n, 1, 1 - z^7) = A_n\,f_n(z) \tag{46}$$

The functions $f_n(x)$ are orthogonal with respect to z^5 as weight factor, that is

$$\int_0^1 z^5\, f_m(z)\, f_n(z)\, dz = 0, \text{ if } m \neq n \tag{47}$$

and

$$\int_0^1 z^5\, f_n^2\, dz = \int_0^1 z^5 \left[F(a_n, \frac{6}{7} - a_n, 1, 1 - z^7) \right]^2 dz = C_n^2 \tag{48}$$

For application to equation (21-a), it is convenient to define a new set of functions $\Phi_n(z)$, for the solutions of the approximate equation (23) such that

$$\Phi_n(z) = \frac{f_n(z)}{C_n\sqrt{\lambda_n}} \tag{49}$$

and

$$\lambda_n \int_0^1 z^5 \, \Phi_m (z) \, \Phi_n (z) \, dz = \delta_{mn} \tag{50}$$

where

$\delta_{mn} = 0$, if $m \neq n$; and

$\delta_{mn} = 1$, if $m = n$

Now, if we assume a solution of the form

$$g(z) = \sum_{n=1}^{S} B_n \, \Phi_n (z) \tag{51}$$

and substitute this expression into equation (21-a), we obtain

$$\sum_{n=1}^{S} B_n \left[\frac{d}{dz} (1 - z^7) \frac{d\Phi_n}{dz} + \omega_n z^7 \Phi_n \right] = 0 \tag{52}$$

and because the Φ_n's satisfy equation (23), this becomes

$$\sum_{n=1}^{S} B_n \left[\lambda_n z^5 \Phi_n (z) - \omega_n z^7 \Phi_n (z) \right] = 0 \tag{53}$$

Thus equation (51) is the desired solution to equation (21-a).

To find the coefficients B_n, we require that the left hand side of this equation be orthogonal to the Φ_n for $n = 1, \ldots, S$.

This implies the following,

$$\sum_{n=1}^{S} B_n (\delta_{mn} - \omega_n A_{mn}) = 0; \quad m = 1, \ldots, S \tag{54}$$

where

$$A_{mn} = \int_0^1 z^7 \Phi_m (z) \Phi_n (z) dz \tag{55}$$

The characteristic equation of the system (54) is

$$\left| \delta_{mn} - \omega_n A_{mn} \right| = 0 \tag{56}$$

and the roots of equation (56) give approximations to the first S eigenvalues of equation (21-a). Further, with each ω_n there is associated a set of B's, and linear combination $\sum_{n=1}^{S} B_n \Phi_n (z)$ is an approximation to the corresponding eigenfunctions of equation (21-a).

V. NUMERICAL SOLUTION FOR THE SPECIAL CASE $\alpha = 0.7$

The specific value of α to be introduced at this point is taken to be that corresponding to the conditions in the airstream at the inlet to the stagnation chamber of the ARL 30-inch hypersonic wind tunnel. For the special case treated in this report, a specific set of conditions has been chosen which corresponds to tunnel operation at approximately Mach 20. Therefore, the conditions to be used in the subsequent numerical analysis to describe the gas at the pipe inlet are given below, while the derivation of the coefficients for free convection, outer surface radiation and conduction from insulation will be shown in the next section. The conditions at the pipe inlet are the following:

P_{inlet} = pressure at pipe inlet = 2000 psia

T_{inlet} = temperature at pipe inlet = $4000^{\circ}R$

$\dot{m}$ = air mass flow = 0.53 lb mass/sec.

A = inner cross-sectional area of pipe = 3.068×10^{-3} ft.^{2}

μ = dynamic viscosity = $480 \times 10^{-7} \dfrac{\text{lb mass}}{(\text{sec})(\text{ft})}$

C_p = specific heat = $0.30 \dfrac{\text{B Tu}}{(\text{lb})(^{\circ}\text{R})}$

ρ = air density = 1.35 lb/ft^3

V = average airstream velocity = 128. ft/sec

Re = Reynolds number = 2.2515×10^5

For $\alpha = 0.7$, the first four values of "a" which satisfy equation (44) are

$$
\begin{aligned}
a_1 &= 1.2862 \\
a_2 &= 2.4374 \\
a_3 &= 3.5195 \\
a_4 &= 4.5695
\end{aligned}
\qquad (57)
$$

Using these values of "a", one can now obtain from equation (45) the values of λ_n corresponding to the values a_1 --- a_n. Also the eigenfunctions

f_n for equation (23) may be obtained by making use of equation (46). Making use of the orthogonality relationship given by equation (48), we can obtain the coefficients C_n which are used in equation (49) to obtain the Φ_n's. We can now proceed to obtain the eigenfunctions g_n (z) by use of equation (51). But before equation (51) can be solved explicitly for g_n (z) the eigenvalues ω_n and coefficients B_n must be found. These can be obtained by solving equation (53). Because of the orthogonality requirement stipulated by equation (53), use of equations (54) and (56) must be made to obtain the coefficients B_n and ω_n. The roots of equation (56) give approximations to the first "S" eigenvalues. Making use of these eigenvalues and equation (54), we can obtain the coefficients B_n. With the coefficients B_n and Φ_n calculated, we can now find the eigenfunctions g_n from equation (51). Before the complete solution for θ can be expressed, the constant A_n must be solved explicitly by making use of equation (19). Having obtained the coefficients A_n, we can now write the complete solution for θ as given by equation (18).

Using the values of "a", and the procedure as outlined in the above paragraph, we can now obtain the eigenfunctions g_n (z) of the original equation (21 - a) in terms of the eigenfunctions Φ_n (z) of equation (23). Making use of the IBM 1620 and 7090 high speed computers, there resulted the following ω_n and g_n (z) when $\alpha = 0.7$.

$$\omega_1 = 30.689$$

$$g_1 = 0.99830\ \Phi_1 + 0.05450\ \Phi_2 - 0.00988\ \Phi_3 + 0.00418\ \Phi_4$$

$$\omega_2 = 232.436$$

$$g_2 = -0.05431\ \Phi_1 + 0.97860\ \Phi_2 + 0.19690\ \Phi_3 - 0.02516\ \Phi_4$$

$$\omega_3 = 581.542$$

$$g_3 = 0.01776\ \Phi_1 - 0.17540\ \Phi_2 + 0.92100\ \Phi_3 + 0.34750\ \Phi_4$$

$$\omega_4 = 1196.131$$

$$g_4 = -0.01249\ \Phi_1 + 0.09104\ \Phi_2 - 0.33610\ \Phi_3 + 0.93730\ \Phi_4 \qquad (58)$$

In the special case where θ_0, the initial temperature distribution, is taken as a constant, the solution expressed analytically by equation (18) can now be written as the following:

$$\frac{\theta}{\theta_0} = \begin{aligned} &1.286 \exp\left(\frac{-\beta_1^2}{pR}x\right)\left[0.99830\ \Phi_1 + 0.05750\ \Phi_2 - 0.00988\ \Phi_3 + 0.00418\ \Phi_4\right] \\ &-0.399 \exp\left(\frac{-\beta_2^2}{pR}x\right)\left[-0.05431\ \Phi_1 + 0.97860\ \Phi_2 + 0.19690\ \Phi_3 - 0.02516\ \Phi_4\right] \\ &+0.265 \exp\left(\frac{-\beta_3^2}{pR}x\right)\left[0.01776\ \Phi_1 - 0.17540\ \Phi_2 + 0.92100\ \Phi_3 + 0.34750\ \Phi_4\right] \\ &-0.217 \exp\left(\frac{-\beta_4^2}{pR}x\right)\left[-0.01249\ \Phi_1 + 0.09104\ \Phi_2 - 0.336100\ \Phi_3 + 0.93730\ \Phi_4\right] \end{aligned} \qquad (59)$$

where the $\frac{\beta_n^2}{pR}$ are as follows:

$$\begin{aligned} \frac{\beta_1^2}{pR} &= 0.3906 \\ \frac{\beta_2^2}{pR} &= 2.9583 \\ \frac{\beta_3^2}{pR} &= 7.4015 \\ \frac{\beta_4^2}{pR} &= 15.2236 \end{aligned} \qquad (60)$$

The eigenfunctions Φ_n are shown as a plot of z versus Φ in Figures 2, 3, 4 and 5.

VI. CALCULATION OF TEMPERATURE DISTRIBUTIONS AS FUNCTIONS OF RADIUS AND AXIAL DISTANCE DOWNSTREAM

1. Introduction

Once the explicit solution for temperature difference "θ" is obtained from equation (59), we can proceed to find actual temperatures as functions of radius and distance. It is more useful to the engineer to have the results expressed as temperatures rather than temperature differences.

To obtain the temperature as a function of location, all three modes of heat transfer should be considered simultaneously. In view of the different laws governing the three modes, namely: conduction, radiation, and convection, an exact solution is extremely difficult if not impossible. In the situation of heat transfer through pipe insulation, one can obtain reasonable results by treating all the above named processes independently. We wish to give a simplified engineering approach for the solution of such a problem by making use of the data obtained in the previous section. The stipulation that the temperatures and temperature gradients be continuous between any two surfaces exchanging heat is made in obtaining the final results.

2. Simplified Engineering Analysis

By requiring that the temperature and temperature gradient be continuous between any two surfaces exchanging heat, we are able to obtain analytical expressions for the centerline, inner and outer wall temperature distributions explicitly. The geometry of the cylindrical chamber cross section being considered herein is made up of n - concentric layers of different insulating materials of various thicknesses as shown in Figure 6. The procedure for matching the regional temperature gradients

and temperatures between any two surfaces exchanging heat will be described in the next sub-section.

3. Matching Techniques

With a known expression for "θ", and a given reference temperature, the appropriate formulae needed for obtaining surface temperatures will now be derived. At the inner wall shown in figure six as region (2), we proceed to equate the temperatures and temperature gradients of the fluid tangent to the solid wall and the wall itself. This can be expressed analytically by the following;

$$\begin{aligned} Aq_{gw} &= Aq_{s_2} \\ BT_{gw} &= BT_2 \\ \hline Aq_{gw} + BT_{gw} &= Aq_{s_2} + BT_2 \end{aligned} \tag{6-1}$$

Now, the heat flux q_{gw} can be expressed by the following equation,

$$q_{gw} = h_{gR}(T_c - T_{gw}) = h_g(T_c - T_2) = h_{gR}\theta \tag{6-2}$$

with h_{gR} defined in appendix B.

Rewriting equation (6-2) as,

$$q_{gw} - h_g(T_c - T_{gw}) = o \tag{6-3}$$

and also after subtracting BT_c from both sides of equation (6-1), one obtains,

$$Aq_{gw} + B(T_{gw} - T_c) = Aq_{s_2} + B(T_2 - T_c) \tag{6-4}$$

Equating coefficients A and B from equation (6-4) to those in equation (6-3), one finds that,

$$A = 1 \text{ and } B = -h_g$$

Therefore,

$$q_{s_2} + h_g (T_c - T_2) = o \tag{6-5}$$

From the solution of the steady state conduction problem, we have the following familiar flux equation,

$$q_{s2} = - \frac{k_{eff} \left(T_2 - T_1\right)}{r_2 \ln \left(\dfrac{r_2}{r_1}\right)} = \frac{-\eta}{r_2} (T_2 - T_1) \tag{6-6}$$

From equations (6-5) and 6-6), we obtain the following important relationship between forced convection and overall conduction,

$$\frac{\eta}{r_2} (T_2 - T_1) = h_g (T_c - T_2); \tag{6-7}$$

or at region (2) we have the temperature difference "θ" expressed as

$$(T_2 - T_1) = \frac{h_g r_2}{\eta} (T_c - T_2). \tag{6-8}$$

The same analogy can be used to obtain expressions for temperature relationships between heat conduction, free convection and radiation at the outer surface of the pipe. Equating the gradients and temperatures of the outer surface of the pipe with a given reference temperature some distance away from the surface, one can use the equations for continuous flux and temperature similar to equation (6-1)

$$\begin{aligned} Aq_{s_1} &= Aq_a \\ BT_1 &= BT_a \end{aligned} \tag{6-9}$$

The flux q_a immediately tangent to the outer surface of the cylinder, can be expressed as,

$$q_a - \bar{h} (T_a - T_{ref}) = o \tag{6-10}$$

Performing the same type of manipulation here as for obtaining equation (6-5), we obtain the formula for external wall flux,

$$q_{s_1} - \bar{h} \ (T_1 - T_{ref}) = 0 \tag{6-11}$$

But we know that the expression for q_{s_1} is

$$q_{s_1} = - \frac{k_{eff}}{r_1 \ \ln\left(\frac{r_2}{r_1}\right)} \ (T_2 - T_1) = \frac{-\eta}{r_1} \ (T_2 - T_1) \tag{6-12}$$

Substituting for q_{s_1} from equation (6-12) into (6-11), we obtain the following relationship,

$$(T_2 - T_1) = \frac{r_1 \ \bar{h}}{\eta} \ (T_1 - T_{ref}). \tag{6-13}$$

If equations (6-8) and (6-13) are now combined, we obtain

$$r_2 \ h_g \ (T_c - T_2) = r_1 \ \bar{h} \ (T_{ref} - T_1) \tag{6-14}$$

or

$$\frac{r_2 \ h_g}{r_1 \ \bar{h}} \ \theta = T_{ref} - T_1 \tag{6-15}$$

We wish to express the temperatures T_1, T_2, and T_c as function of the parameters h_g, $\bar{h}$ and η as well as the radii r_1, r_2, and temperature difference θ. These relationships in algebraic form are,

$$T_1 = T_{ref} - \left(\frac{r_2}{r_1}\right)\left(\frac{h_g}{\bar{h}}\right)\theta \tag{6-16}$$

$$T_3 = T_1 - r_2 \left(\frac{h_g}{\eta}\right) \theta \tag{6-17}$$

$$T_c = \theta + T_2 \tag{6-18}$$

After some further manipulation of equations (6-16), (6-17) and (6-18), we obtain the following useful formulae for the temperature.

$$T_c = T_{ref} + \theta\left[1 - r_2 h_g\left(\frac{1}{r_1 \bar{h}} - \frac{1}{\eta}\right)\right] \qquad (6\text{-}19)$$

or

$$T_c = T_{ref} + \theta\left[1 - \left(\frac{r_2}{r_1}\right)\left(\frac{h_g}{\bar{h}}\right)\left(\frac{1}{\eta}\right)(\eta - r_1 \bar{h})\right] \qquad (6\text{-}20)$$

and denoting the portion in the brackets as coefficient "c_1" we have

$$T_c = T_{ref} + c_1 \theta \qquad (6\text{-}21)$$

We can also express the temperatures T_2 and T_1 as follows,

$$T_2 = T_{ref} - c_2 \theta \qquad (6\text{-}22)$$

where c_2 is,

$$c_2 = \left(\frac{r_2}{r_1}\right)\left(\frac{h_g}{\bar{h}}\right)\left(1 - \frac{r_1}{\eta} \bar{h}\right);$$

and for T_1 the following:

$$T_1 = T_{ref} - c_3 \theta \qquad (6\text{-}23)$$

with c_3 expressed as,

$$c_3 = \left(\frac{r_2}{r_1}\right)\left(\frac{h_g}{\bar{h}}\right)$$

To obtain actual temperatures, the coefficients $\bar{h}$, k_{eff}, etc. must be defined and expressed analytically. For the special case where the value of the heat transfer parameter α was 0.7, the formulae for the coefficients mentioned above are given in the Appendix.The results obtained for the case $\alpha = 0.7$ are presented (Figs. 9-11) as plots of axial temperature distribution along the pipe. Some radial temperature profiles at specific axial locations are also presented (Figs.12-15).

VII. CONCLUDING REMARKS

1. The plot of λ_n, ω_n versus α, shown in Figure 8, indicates that the exponential decay of the temperature along the channel increases steadily, at first rather slowly up to $\alpha = 0.5$ then quite rapidly until the asymptotic value ($\alpha = \infty$) is reached. This trend is the same for equations (21-a) and (23).

2. The accuracy in approximating the uniform radial temperature distribution at the pipe entrance is dependent upon the number of eigenfunctions used. With only four eigenfunctions, the accuracy was poor near the wall but excellent near the centerline. However, the accuracy of the radial distribution at the wall improves considerably at subsequent axial positions.

3. For the numerical example chosen, the fully developed thermal field is attained when the gas has passed through 136 tube diameters. Also, the centerline temperature dropped rapidly from its maximum value in 96 tube diameters, then asymptotically approached a minimum value, while the corresponding radial distribution became uniform.

4. In the axial direction of the fluid downstream, the temperature distribution becomes less and less dependent upon the higher eigenfunctions. This implies that to obtain a first approximation to a very complicated heat transfer problem, one need only to make computations using the first, or at most, first and second eigenfunctions.

5. In using the eigenvalue - eigenfunction approach when obtaining approximations to the solution for the heat transfer equation, some inherent error exists in the (n - 1) and n^{th} terms of the hypergeometric series. The effect of the error from approximating the last two terms is noticed only

when approximating a uniform temperature function at the entrance section.

6. The axial and radial temperature profiles, as presented in Figures 9 through 15 inclusive, may be considered reasonably accurate for engineering purposes. One reason for any inaccuracy in applying these results would be that near the entrance section the velocity profile may not be fully established at the actual thermal entrance section. The analysis contained the assumption that the velocity profile was fully established at the thermal entrance section.

REFERENCES

1. Latzko, H., NACA TM 1068 (1944); original in German.

2. Sleicher, C. A., Jr., and Tribus, M., "Heat Transfer in a Pipe With Turbulent Flow and Arbitrary Wall-Temperature Distribution", Trans., ASME, Vol. 79, 1957.

3. Becker, H. L., "Applied Scientific Research Series A, " 1956.

4. Deissler, R. G., NACA TN 3016 (1953).

5. Sparrow, E. M., Hallman, T. M. and Siegel, R., "Turbulent Heat Transfer in the Thermal Entrance Region of a Pipe with Uniform Heat Flux, " Applied Science Research, Vol. 7, 1957.

6. Durfee, W. H., "Heat Flow and a Fluid with Eddying Flow, " Readers Forum Journal of the Aeronautical Sciences, Vol. 23, pp. 188-189, (Feb 1956).

7. Fettis, H. E., "On the Eigenvalue Latzko's Differential Equation, " ZAMM, Vol. 37, Nos. 9, 10, (Sep/Oct 1957).

8. Jakob, M., "Heat Transfer, " Vol. I, John Wiley and Sons, Inc., New York, (1949).

9. Magnus, W., and Oberhettinger, F., "Formeln Und Sätze Für Die Speziellen Funktionen Der Mathematischen Physik, " Springer (1948).

10. Grober, Erk, and Grigull, "Fundamentals of Heat Transfer, " McGraw-Hill Book Company, Inc., New York, 1961.

APPENDIX

In the heat transfer from the outer wall of the cylinder to a surrounding gaseous medium, e.g. air, free convection and external radiation are the contributing factors in calculating the temperature at the outer surface of the cylindrical wall. The two contributions to the heat transfer due to free convection denoted as h_c and that due to radiation defined as h_r are additive as a first approximation. Therefore, the heat flux q may be written in the following form,

$$q = (h_c + h_r)\ (T_1 - T_{ref}) \tag{a}$$

The resultant coefficient used in section VI as $\bar{h}$ is defined as,

$$\bar{h} = h_c + h_r \ . \tag{b}$$

The formulation used for obtaining the coefficients h_c and h_r respectively are given below

$$\frac{h_c\, D}{k_{ref}} = 0.372\ Gr^{\frac{1}{4}} \tag{c}$$

where,

D = diameter of the cylinder,

G_r = Grashof number

k_{ref} = thermal conductivity of the gas surrounding the outer wall

Equation (c) is now written as,

$$h_c = k_{ref}\ \frac{(0.372)}{D}\ G_r^{\frac{1}{4}}$$

or,

$$h_c = k_{ref}\ \frac{(0.372)}{D} \left\{ g\ \frac{(T_1 - T_{ref})\ D^3}{T_1\ \nu^2} \right\}^{\frac{1}{4}} \tag{d}$$

where the Grashof number is defined as

$$G_r = \frac{g\beta_1 \ \theta_{ref} \ D^3}{\nu^2}$$

$$\beta_1 = \frac{1}{T_1}, \ g = 32.2 \ \frac{ft}{sec^2}$$

and ν is the kinematic viscosity of the surrounding gas. The term θ_{ref} is defined as the temperature difference between the two surfaces exchanging heat.

A further simplification of the coefficient h_c is,

$$h_c = k_{ref} \frac{(0.375)}{D^{\frac{1}{4}}} \left\{ g \frac{(T_1 - T_{ref})}{T_1 \ \nu^2} \right\}^{\frac{1}{4}} \tag{e}$$

The radient heat transfer coefficient "h_r" used in the problem is given by the formula,

$$h_r = \frac{1}{\frac{1}{C_I} + \frac{A_I}{A_{II}} \left(\frac{1}{C_{II}} - \frac{1}{C_b} \right)} \left[\frac{T_1^3 + T_1^2 \ T_{ref} + T_1 \ T_{ref}^2 + T_{ref}^3}{10^8} \right] \tag{f}$$

where,

C = the exchange coefficient = $10^8 \ \sigma \epsilon$

σ = Stephan Boltzmann constant

ϵ = emmisivity

A_I = cylindrical area

A_{II} = surrounding room area

Since the ratio of A_I to A_{II} is small, the term $\frac{A_I}{A_{II}} \left(\frac{1}{C_{II}} - \frac{1}{C_b} \right)$ is neglected.

Now equation (f) can be written as the following

$$h_r = .173 \left(\frac{T_1^3 + T_1^2 \, T_{ref} + T_1 \, T_{ref}^2 + T_{ref}^3}{10^8} \right) \tag{g}$$

Formulae d and f added together gives us the coefficient $\bar{h}$ which was previously defined as

$\bar{h} = h_c + h_r$, can now be written as

$$\bar{h} = \frac{(0.372)\,(k_{ref})}{D^{\frac{1}{4}}} \left\{ \frac{g\,(T_1 - T_{ref})}{T_1 \, \nu^2} \right\}^{\frac{1}{4}}$$

$$+ \; 0.173 \left\{ \frac{T_1^3 + T_1^2 \, T_{ref} + T_1 \, T_{ref}^2 + T_{ref}^3}{10^8} \right\} \tag{h}$$

The standard formula used for the over all thermal conductivity of the solid layers of the cylinder (see figure 7) is

$$K_{m_r} = \frac{1}{K_{m_1}} \ln\left(\frac{D_2}{D_1}\right) + \frac{1}{K_{m_2}} \left(\ln \frac{D_3}{D_2} + \ln \frac{D_4}{D_3} \right) + \tag{i}$$

$$+ \; \frac{1}{K_{m_3}} \ln\left(\frac{D_5}{D_4}\right) + \frac{1}{K_{m_4}} \ln\left(\frac{D_6}{D_5}\right) ,$$

where the thermal conductivities are given for the following materials

K_{m_1} - dense partially stabilized zirconia

K_{m_2} - cubic zirconia

K_{m_3} - fiberfrax

K_{m_4} - steel

To calculate the unit thermal convective conductance h_{gR}, a ratio is set up of the amount of heat transferred per unit length of wall surface for the mixed mean temperature difference at the cross section; that is,

$$h_{gR} = \frac{q_R}{\theta_m} \qquad \text{(j)}$$

From the shear relationship, one can arrive at the following equation for q_R,

$$q_{gw} = 0.176\, V^{\frac{3}{4}}\, \rho\, C_p\, \nu^{\frac{1}{4}} \lim_{\delta \to o} \left[\frac{\partial \theta}{\partial \delta}\, \delta^{\frac{6}{7}} \right] \qquad \text{(k)}$$

where δ represents the radial difference (R - r) in the fluid portion of the cylinder. The average temperature $\theta_m = \frac{1}{\pi R^2} \int_o^R \theta\, 2\, \pi\, r\, d\, r.$ (l)

With the help of aknown temperature distribution (θ), h_g can now be computed anywhere downstream. Putting into the formula for h_g the analytical expression for q_r and θ_m, we obtain.

$$h_{gR} = \frac{\dfrac{0.176\, V^{\frac{3}{4}}\, \rho\, C_p\, \nu^{\frac{1}{4}}}{R^{3\ 28}} \left[\lim_{\delta \to o} \frac{\partial \theta}{\partial \delta}\, \delta^{\frac{6}{7}} \right]}{\dfrac{1}{\pi R^2} \displaystyle\int_o^R \theta\, (r)\, 2\, \pi\, r\, d\, r} \qquad \text{(m)}$$

STAGNATION CHAMBER
ARL 30" HYPERSONIC WIND TUNNEL

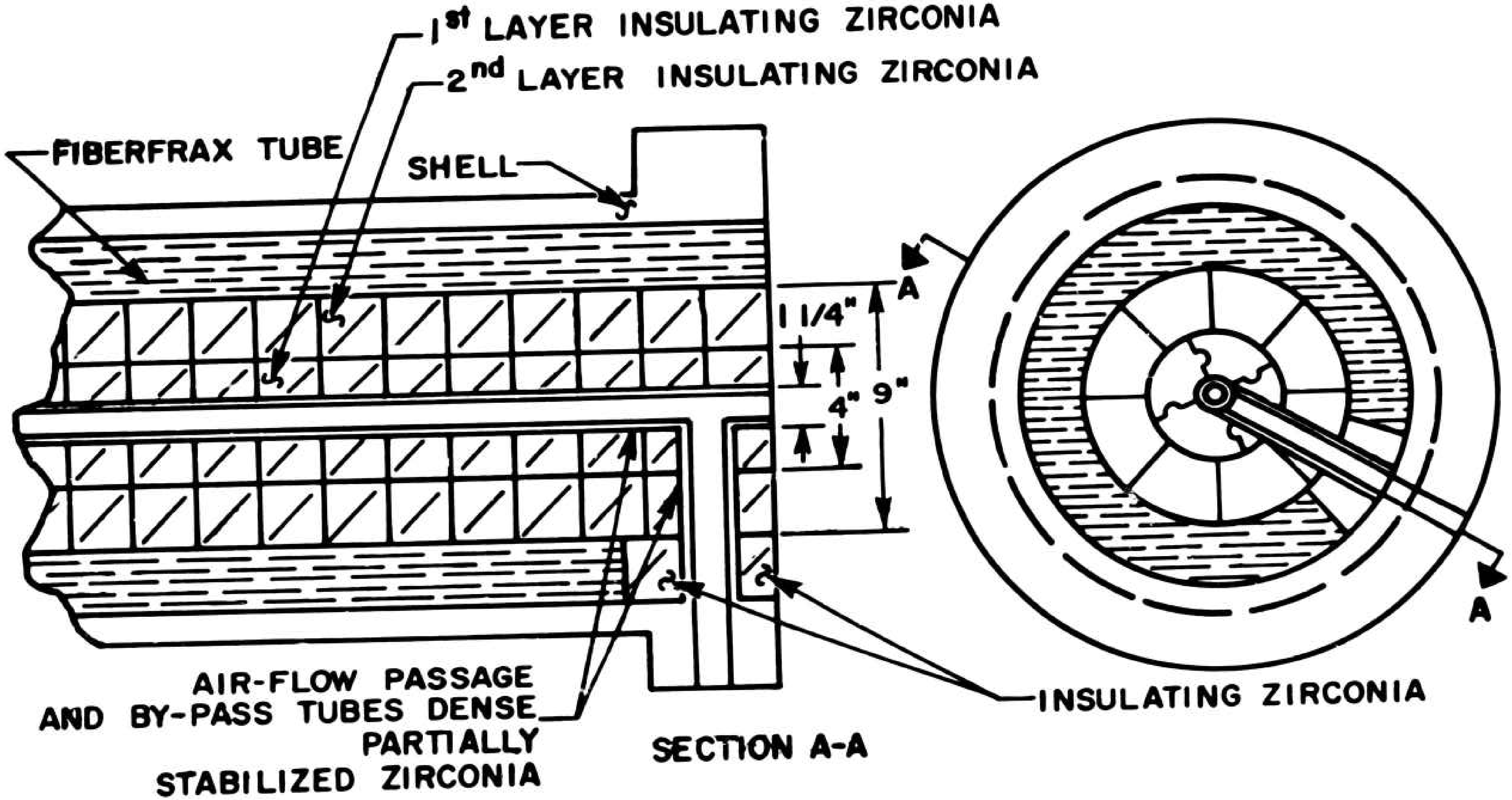


FIGURE 1

FIRST EIGENFUNCTION Φ_1 FOR $\alpha = 0.7$

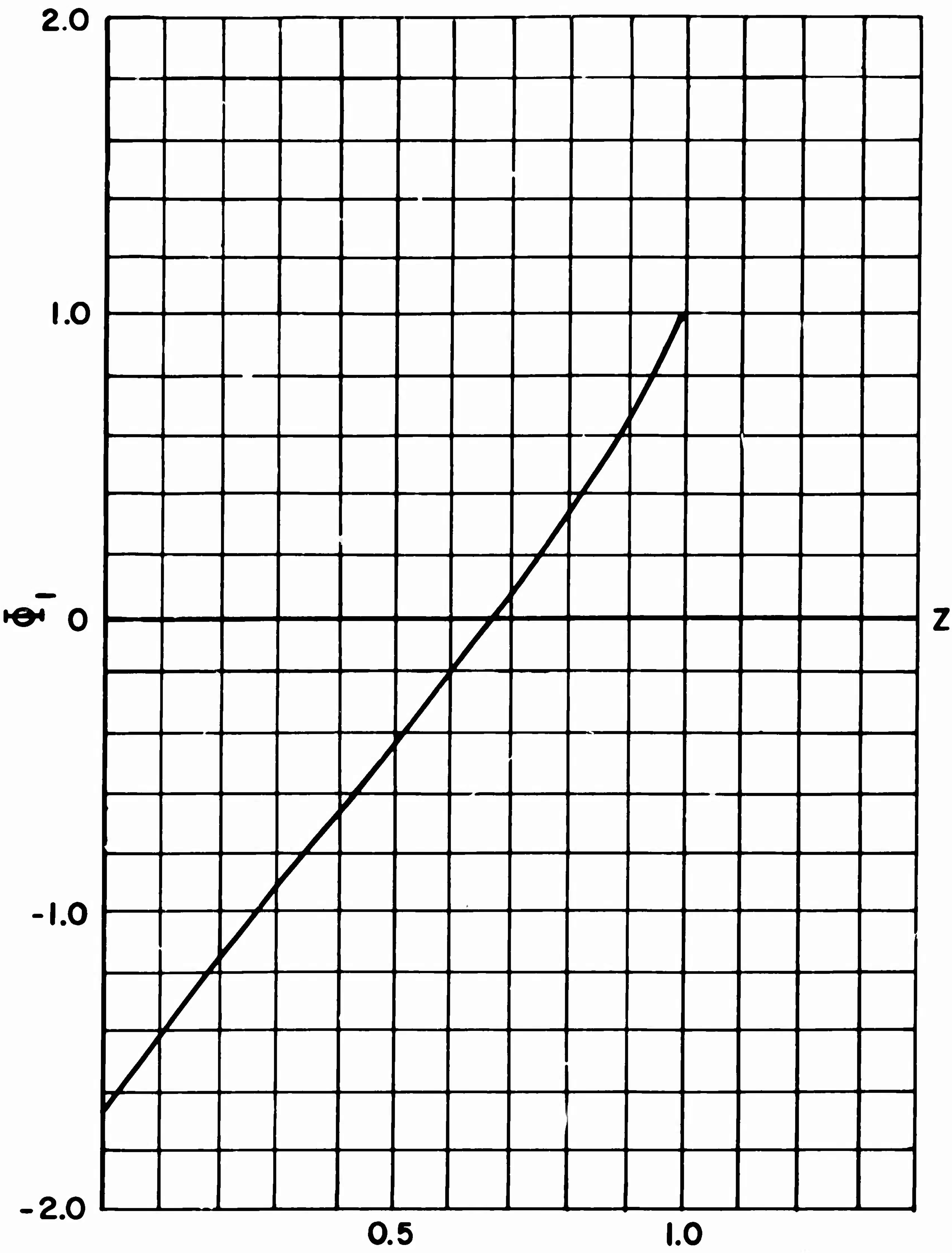


FIGURE 2

SECOND EIGENFUNCTION Φ_2 FOR $\alpha = 0.7$

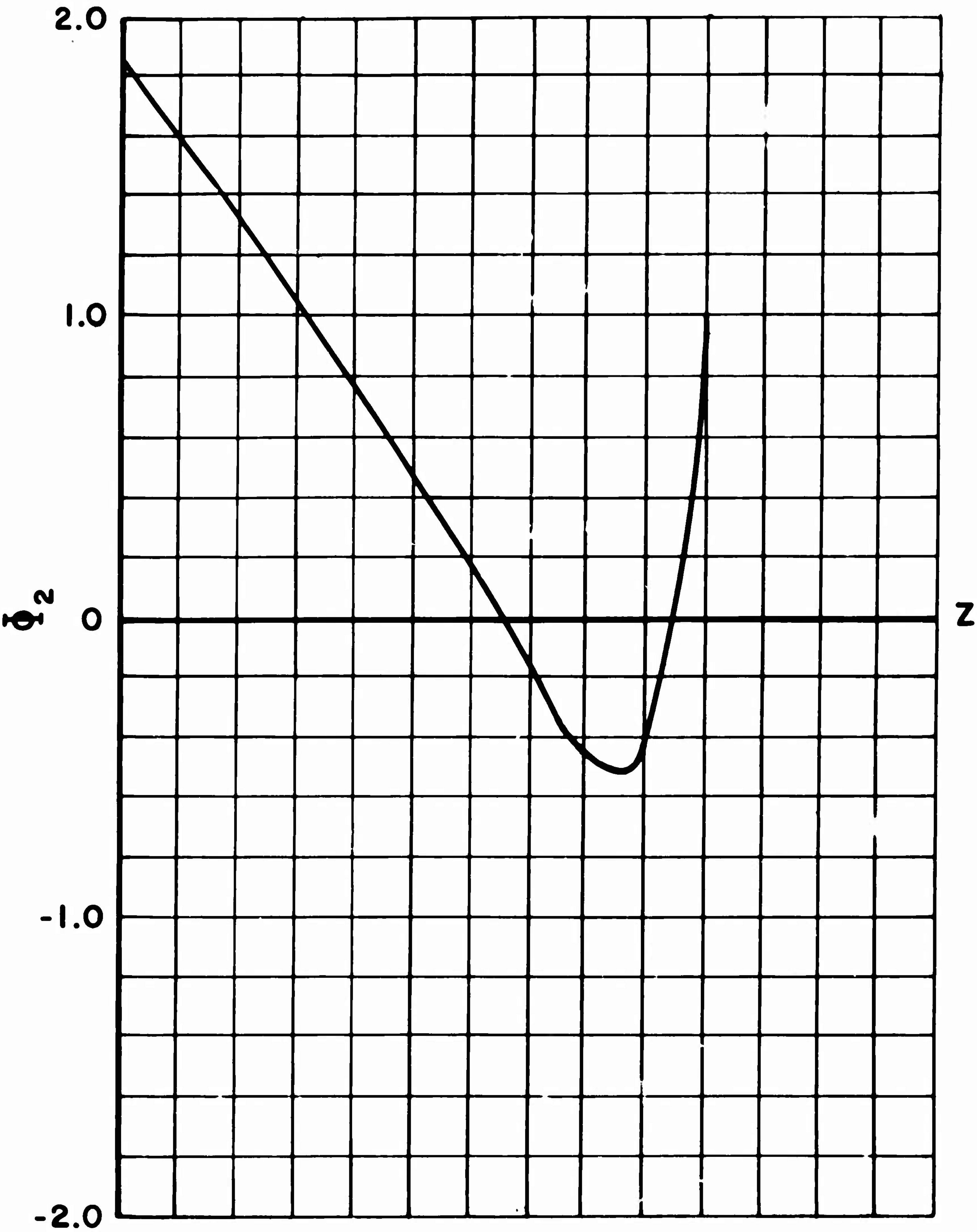


FIGURE 3

THIRD EIGENFUNCTION Φ_3 FOR $\alpha = 0.7$

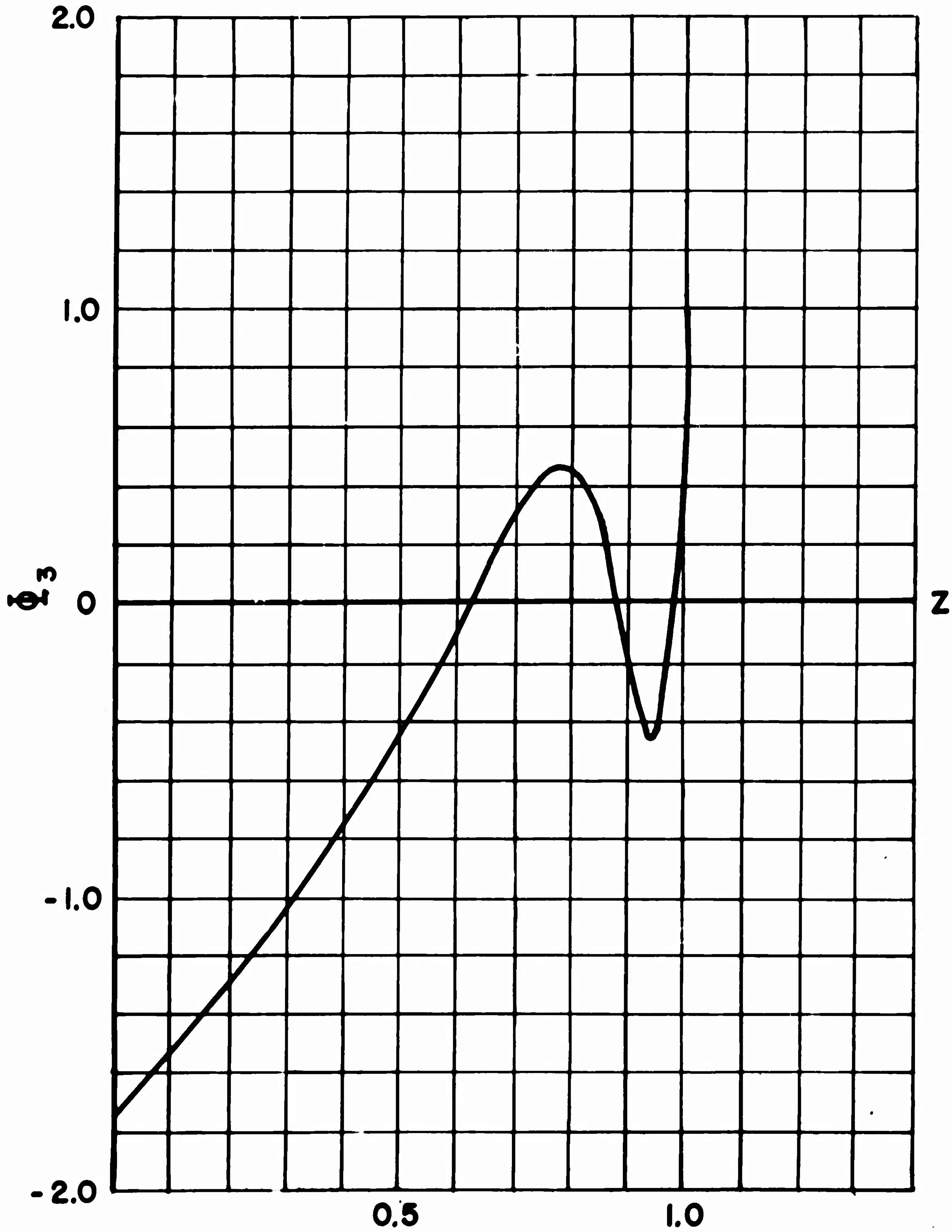


FIGURE 4

FOURTH EIGENFUNCTION Φ_4 FOR $\alpha = 0.7$

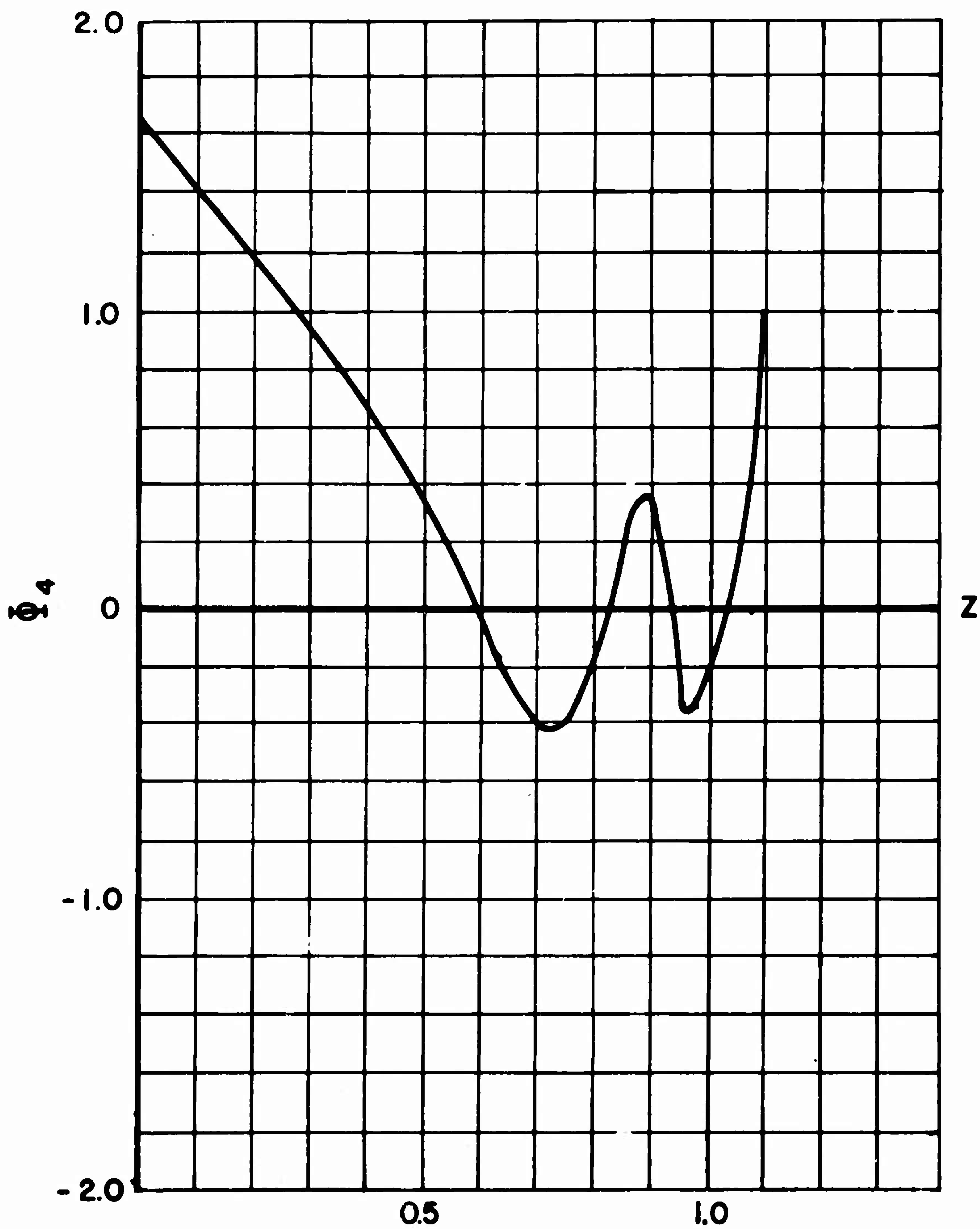


FIGURE 5

PROBLEM GEOMETRY FOR THE SPECIAL CASE $\alpha = 0.7$

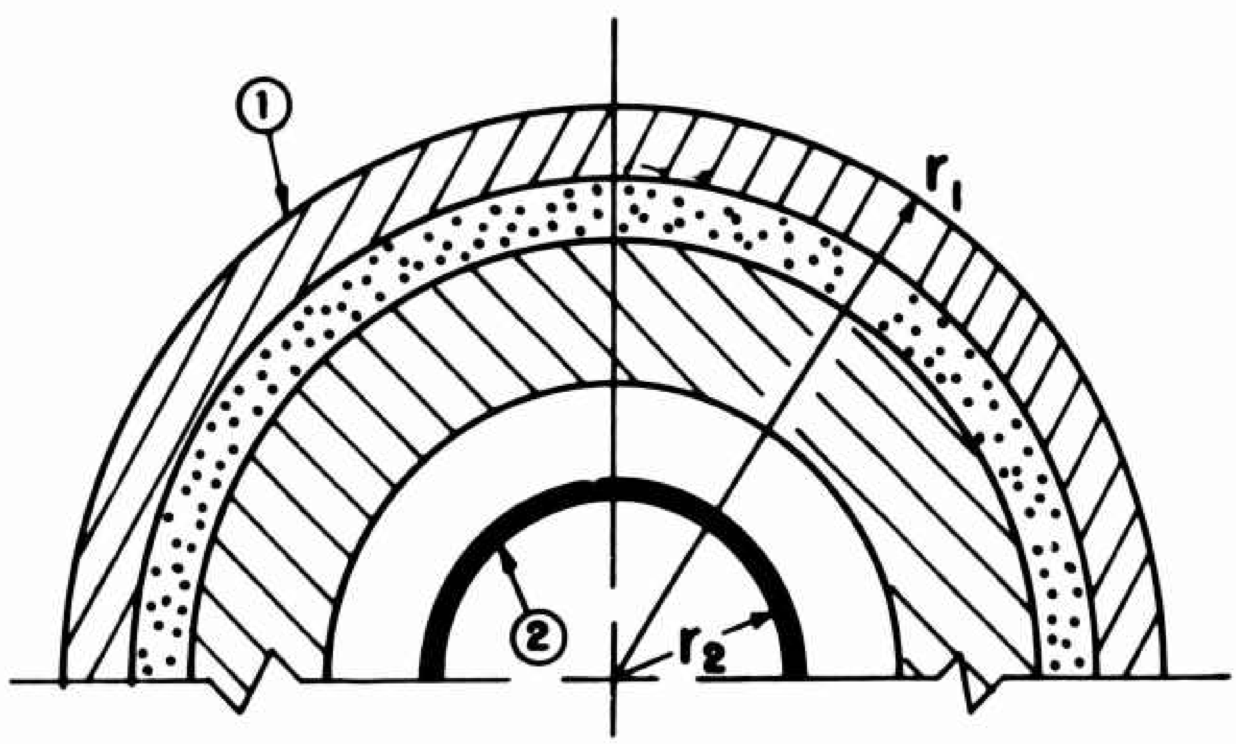


FIGURE 6

DIMENSIONAL CROSS–SECTION OF THE STAGNATION CHAMBER

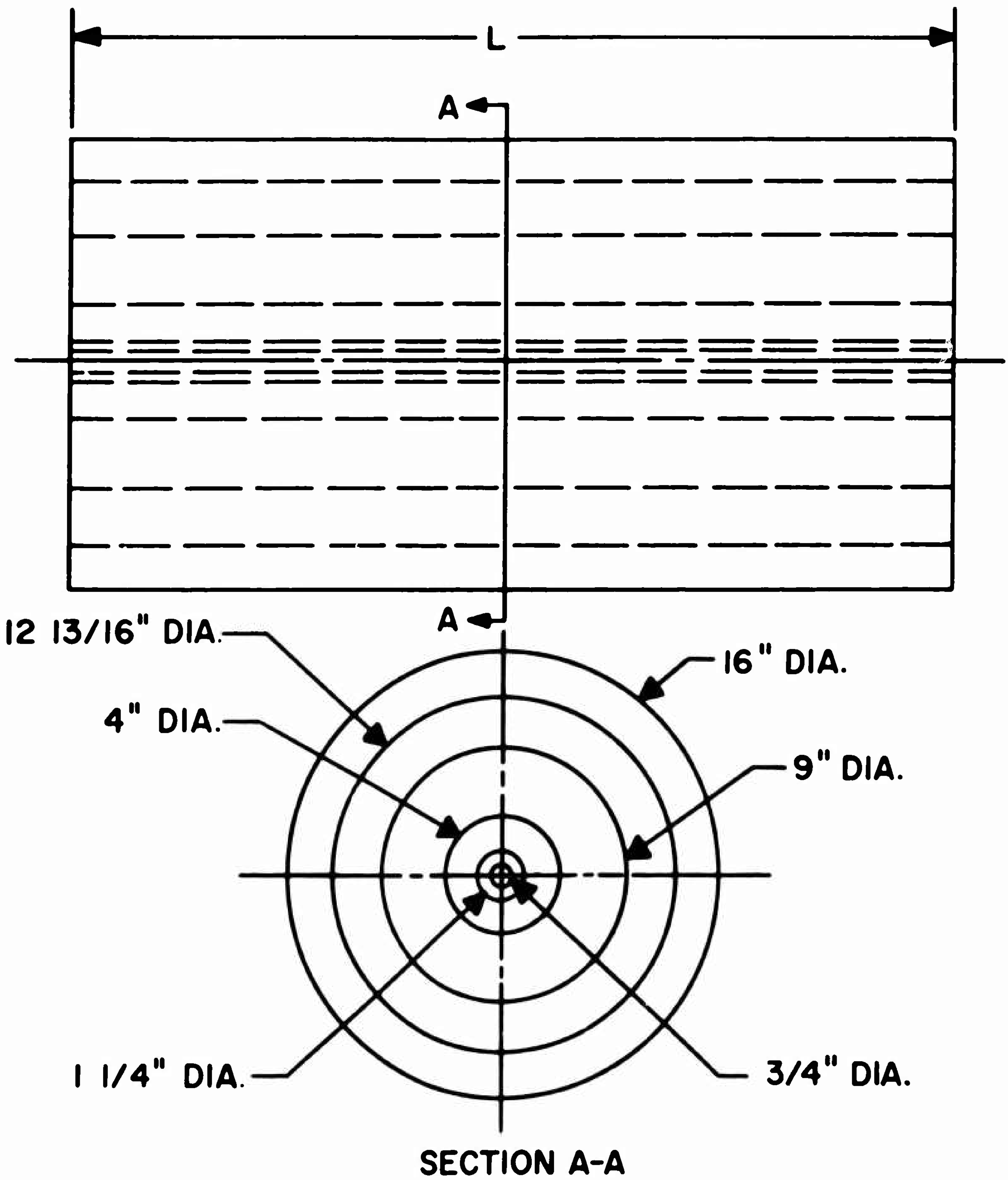


FIGURE 7

COMPARISON OF EIGENVALUES FOR APPROXIMATE AND ACTUAL EQUATIONS AS A FUNCTION OF α

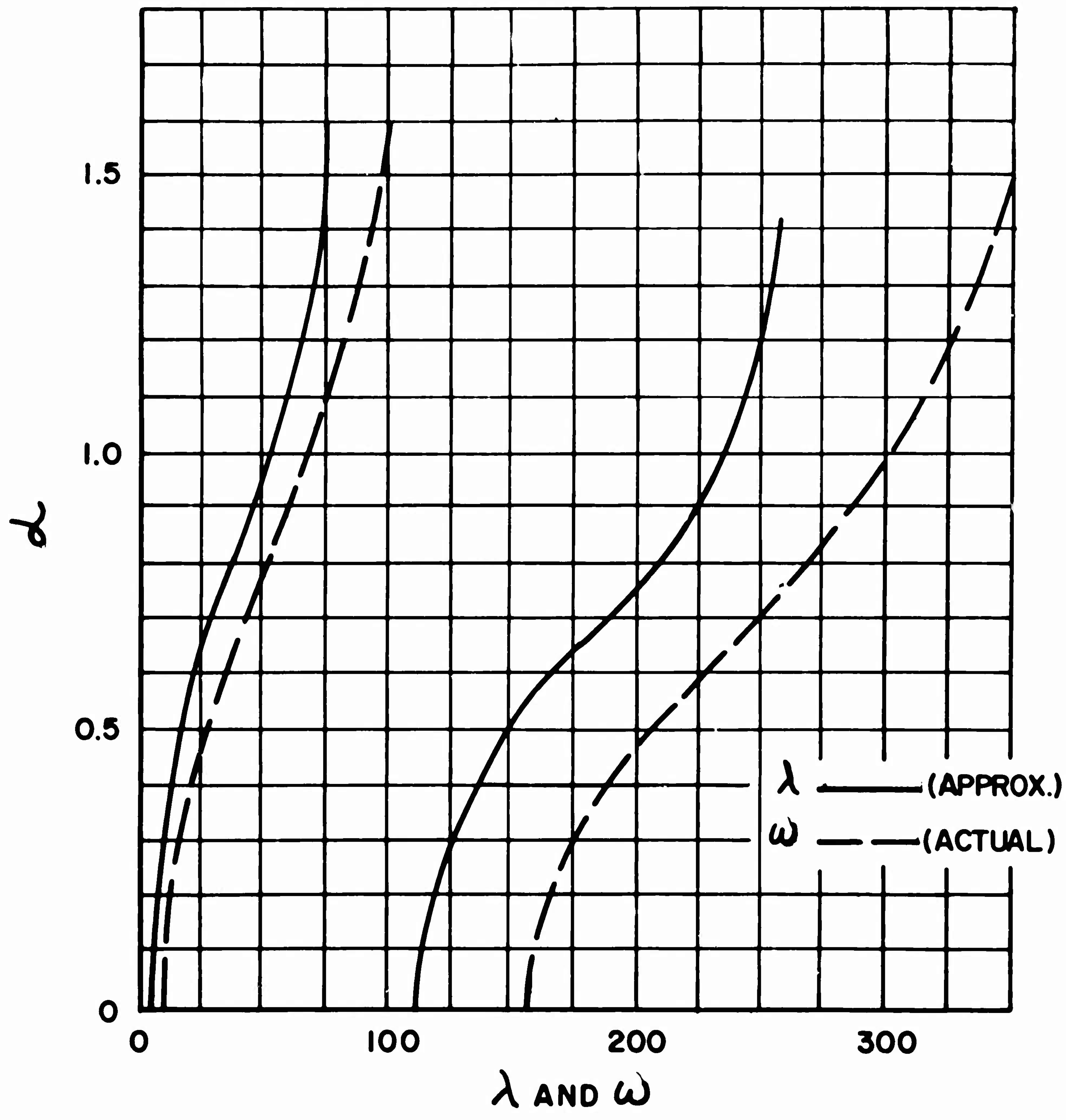


FIGURE 8

EXTERNAL WALL TEMPERATURE DISTRIBUTION VERSUS AXIAL DISTANCE

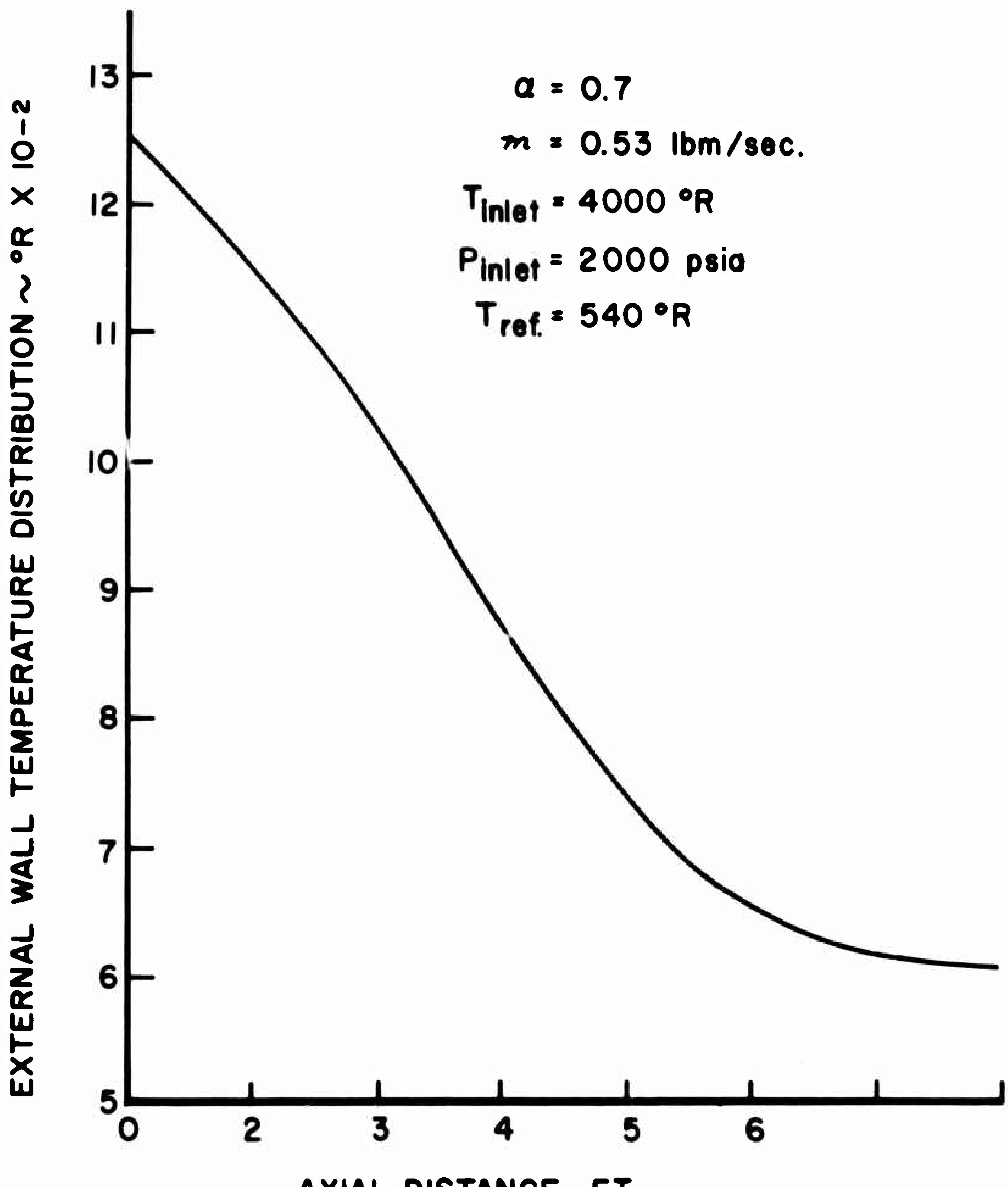


FIGURE 9

INNER WALL TEMPERATURE DISTRIBUTION VERSUS AXIAL DISTANCE

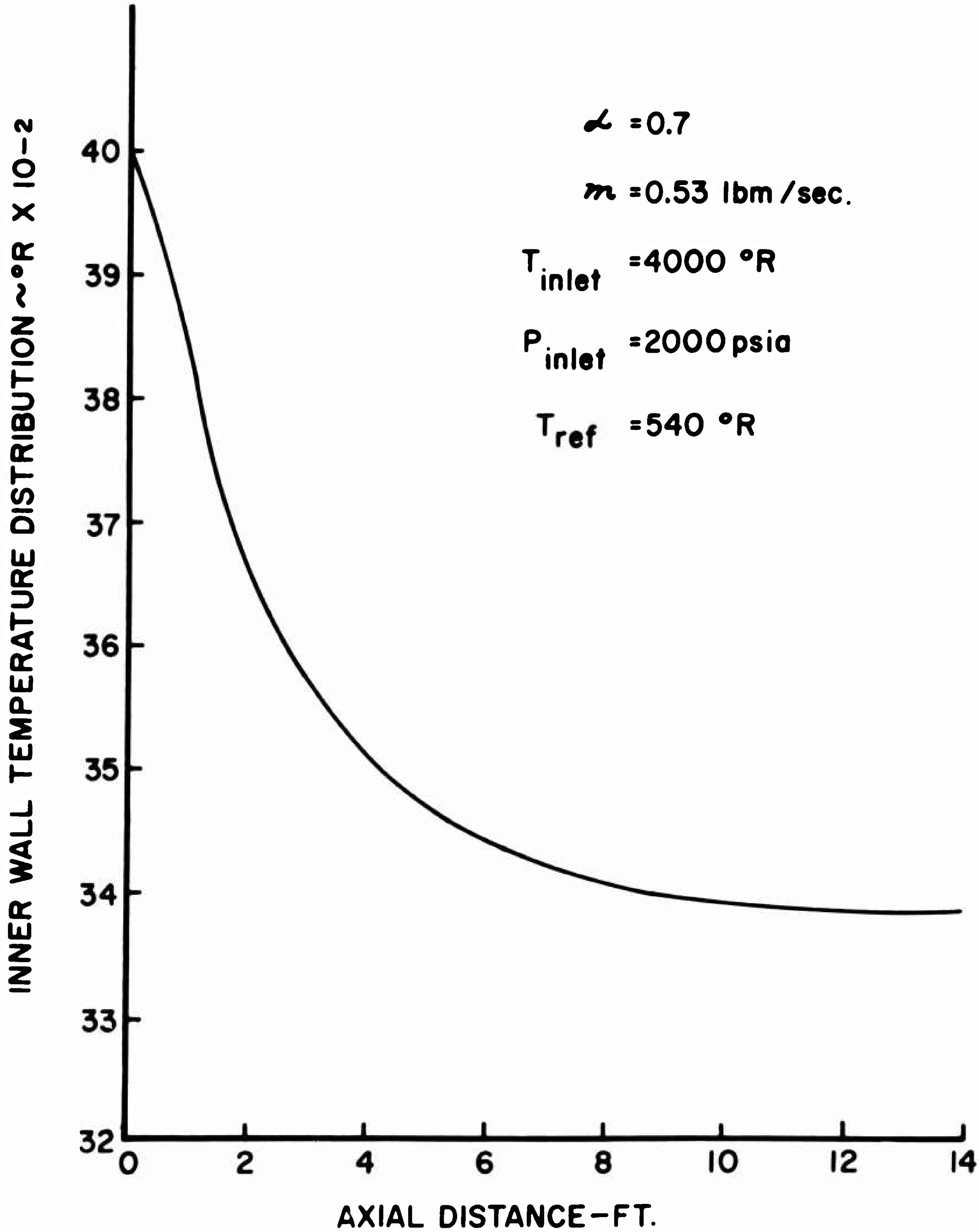


FIGURE 10

CENTERLINE TEMPERATURE DISTRIBUTION VERSUS AXIAL DISTANCE

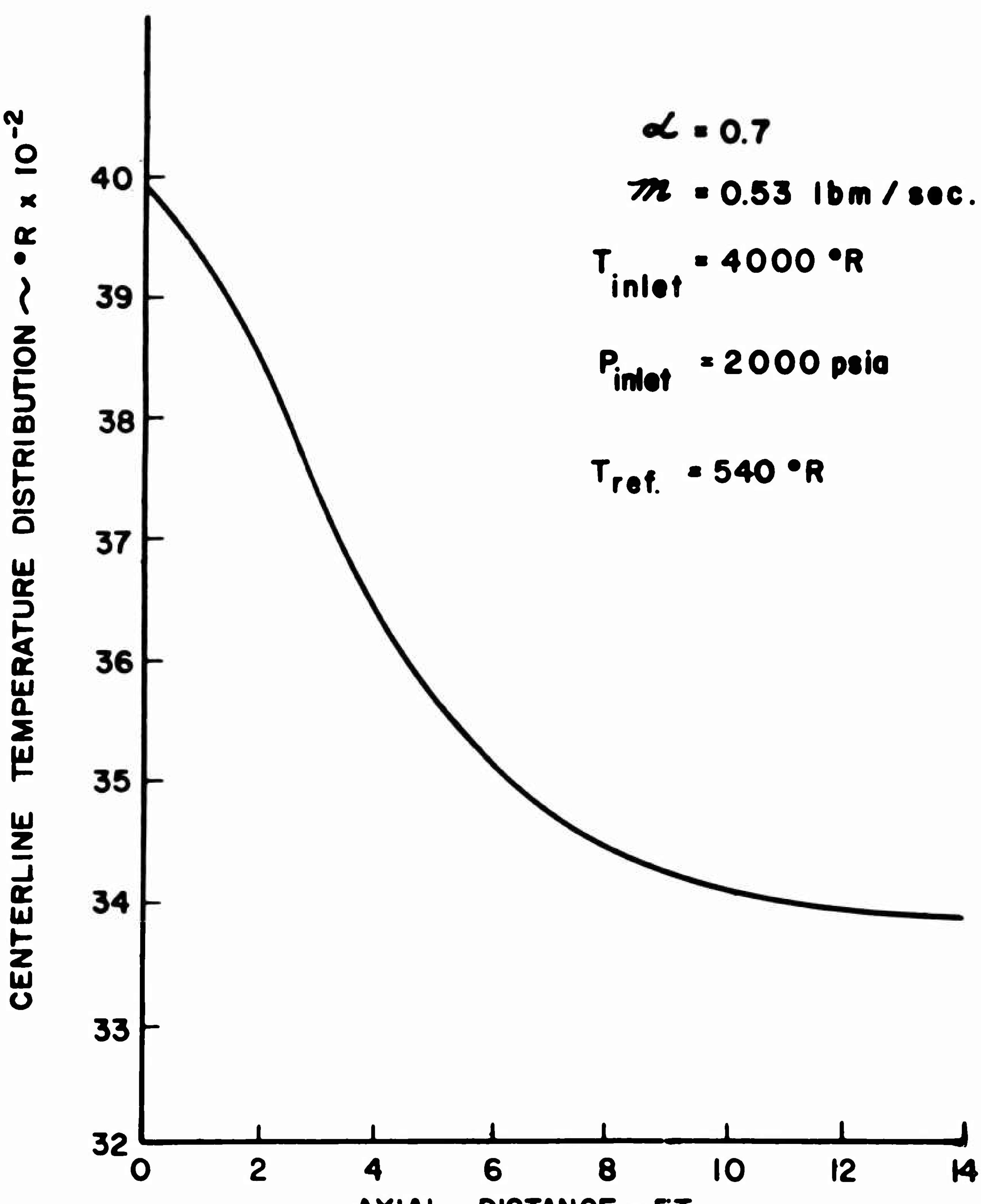
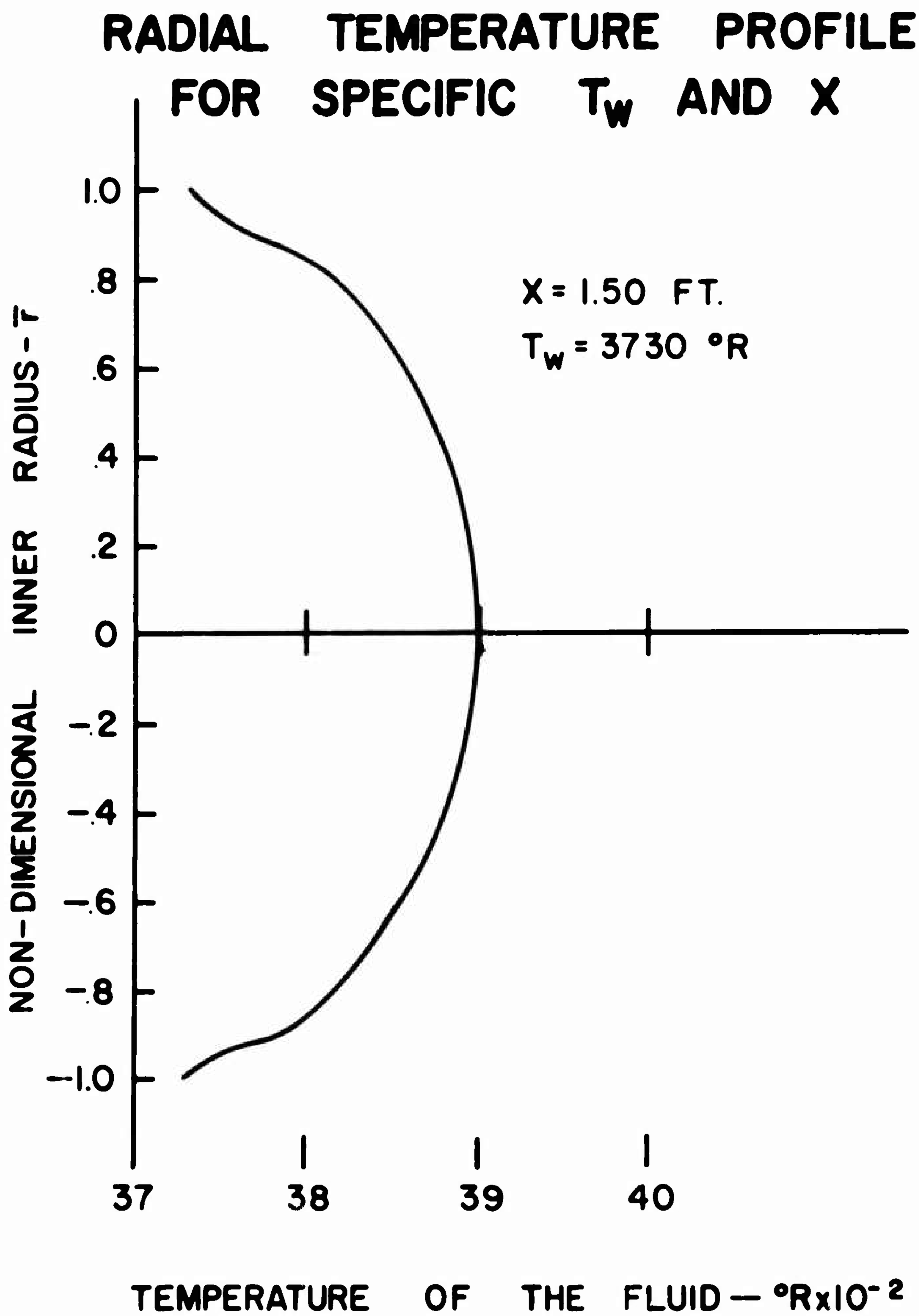


FIGURE 11



RADIAL TEMPERATURE PROFILE
FOR SPECIFIC T_W AND X
X = 1.50 FT.
T_W = 3730 °R
1.0
.8
.6
.4
.2
0
−.2
−.4
−.6
−.8
−1.0
NON-DIMENSIONAL INNER RADIUS - $\bar{r}$
37
38
39
40
TEMPERATURE OF THE FLUID — °R x 10^{-2}

FIGURE 12

RADIAL TEMPERATURE PROFILE FOR SPECIFIC T_W AND X

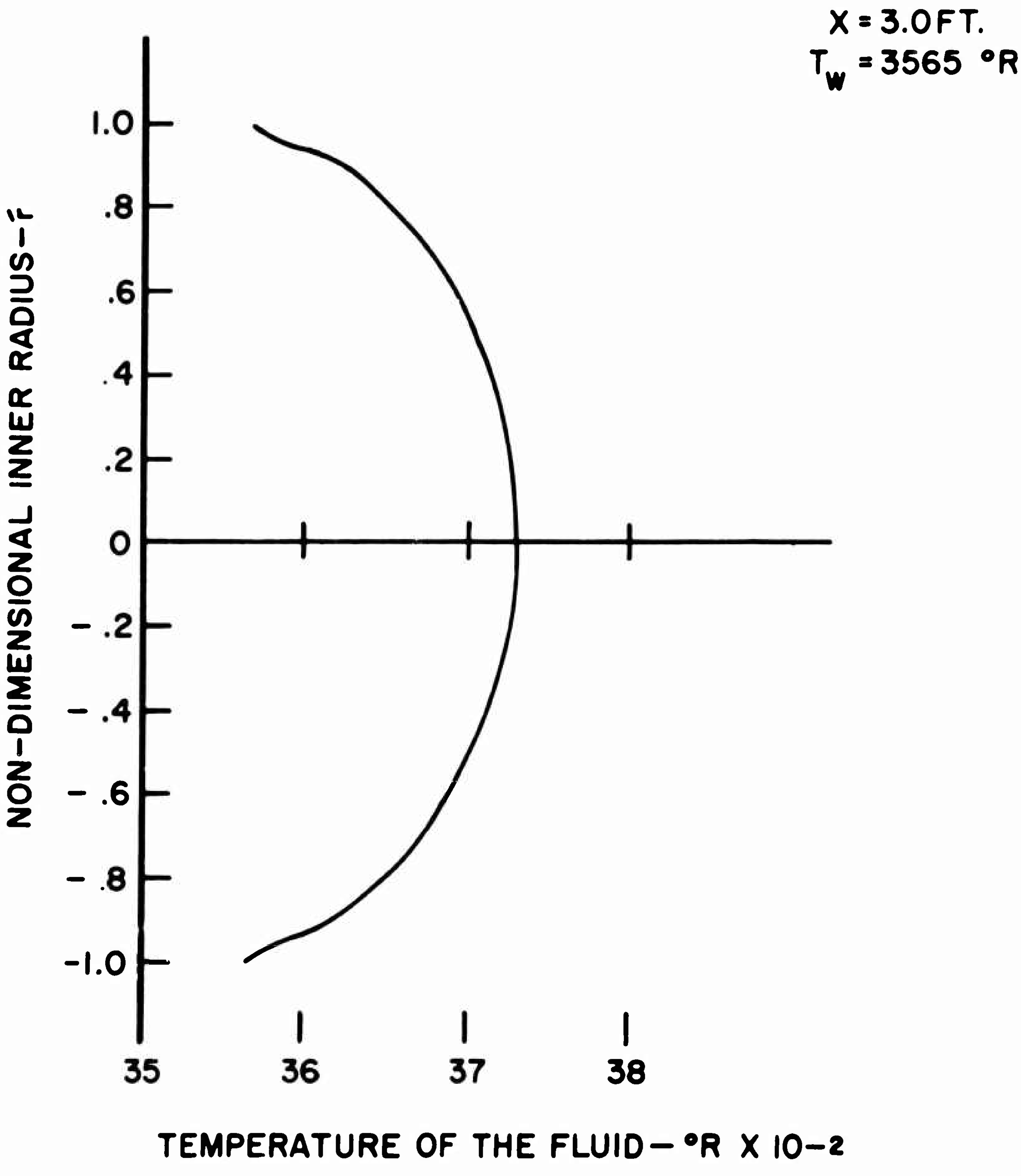


FIGURE 13

RADIAL TEMPERATURE PROFILE FOR SPECIFIC T_W AND X

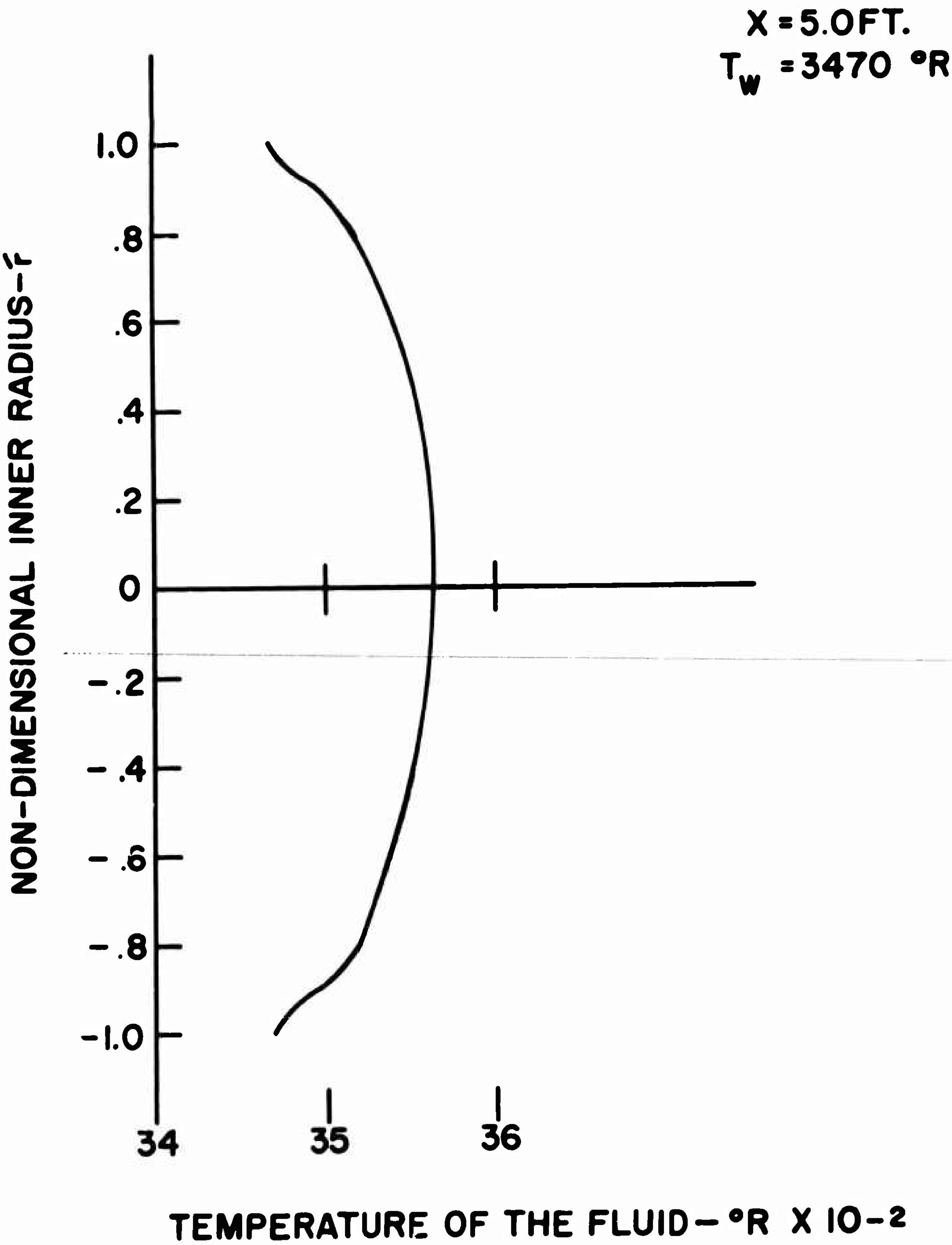


FIGURE 14

RADIAL TEMPERATURE PROFILE FOR SPECIFIC T_W AND X

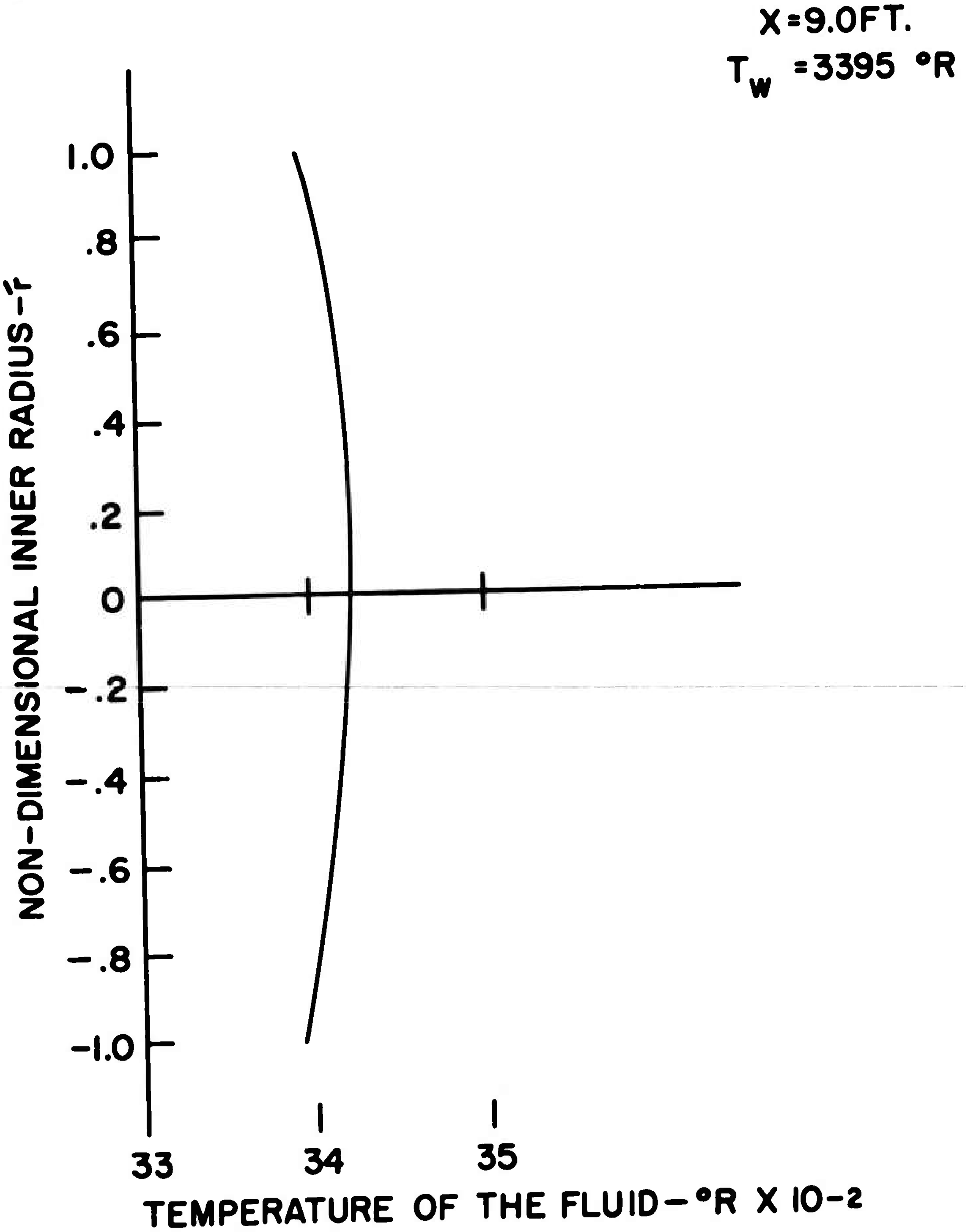


FIGURE 15